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OF THE

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TECHNICAL PAPERS

AND

DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

PROBLEMS AND CONTROL OF DECENTRALIZATION IN URBAN AREAS

A SYMPOSIUM

PAGE

Problems of United States Cities.
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NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **April 1, 1948.**

PROBLEMS OF UNITED STATES CITIES

By John M. Picton,[1] M. ASCE

Effective action is needed to help cities face the multitude of problems presented by decentralization. The spread of residential areas beyond city limits, whereby people seek to enjoy the benefits of the city without paying their just share of the taxes, poses problems for the future as well as for the present. A diagnosis of the present situation of United States cities and some explanation of what these problems are is given in this paper; the one that follows offers suggested approaches to the solution of these problems.

Much thoughtful attention has been given to the problems of cities during recent decades. Among other studies on this subject the good and bad characteristics of urban areas in the United States have been thoroughly explored in reports by the National Resources Committee, and more recently by the National Interregional Highway Committee.

Until recent years, however, most cities have had no adequate over-all plans for future development. Narrow streets built for horse and buggy use are unable to cope with current traffic demands, and traffic circulation systems are inadequate because of indirect, narrow routes. Mass transportation systems with outmoded, early day patterns and inadequate equipment did a remarkable job during World War II. However, because of the inconveniences of public transit a vicious cycle will cause an increase in the use of private vehicles, with more and more congestion of narrow streets having inadequate off-street parking facilities.

Little or no attention has been given to the appropriate development of waterfront areas. Industrial areas are clogged by the conflict of street traffic with railroad traffic, these areas often being choked by the rail facilities which originally led to their development. In addition, piecemeal development of acreage tracts without proper guidance or control has resulted in stereotyped gridiron patterns with dead-end streets, offset streets, and poorly laid out subdivisions. In many instances these layouts have ignored topography and have not resulted in appropriate home sites.

Uneconomic patterns of land use have resulted from indiscriminate mixtures of industrial, business, multifamily dwelling, and residential uses in older sections of cities. Newer sections are generally better developed but have too much area zoned for business use, too much vacant land, and too little recreational and other community facilities. Delightful environments are found in isolated sections of small neighborhoods, but these usually are too small and tend to be crisscrossed by major traffic arteries. There has been a decline of the central business districts; more and more shoppers prefer to trade in the outlying business areas. As a result the stability of the downtown areas has been undermined.

Many cities during the decade from 1930 to 1940 suffered losses in population, whereas a few showed population gains. Nevertheless, their metropoli-

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tan areas increased in size because of the "flight to the suburbs." The major part of residential building in the last few years has been beyond the central city limits, requiring the extension of utilities. By contrast, one third of the city areas are still vacant even though they are already provided with sewers, water, paved streets, and utilities. Curtailment of residential building activities during World War II resulted in a critical shortage of housing accommodations. Little relief is in sight until the middle 1950's at the 1946 and 1947 rate of activities.

Over one third of the population is poorly housed in obsolete structures needing major repairs and lacking sanitary facilities. Additional blighted areas are rapidly being created in newer sections and beyond the city limits because of poor initial subdivision layouts, poor septic tank installations, inadequately sized lots, and similar reasons. The congestion and blight which these people sought to avoid in the cities is rapidly appearing in these outlying areas where they had hoped to have permanently more light, air, and open space.

Urban redevelopment bills have been passed in many of the states, and the means are now available for private capital to build redevelopment projects. Nevertheless, persons with deep-seated prejudices spend the major part of their time fighting public housing projects, which are badly needed to accompany private urban redevelopment. Land costs in blighted areas are excessive, requiring high rents or larger subsidies than are available under private urban redevelopment procedures. Only a few redevelopment projects have been started, except where the private developer proposed to house many times the present population and thus, by means of vertical expansion, to hopelessly overcrowd the land. Undoubtedly this will result in an extensive redevelopment program in the 1960's and 1970's to rebuild the slum clearance structures whose construction is being attempted in the 1940's and 1950's.

It is generally accepted that cities cannot continue without end to widen major streets to accommodate an ever-increasing flow of traffic. Express highways seem to be the answer to this pressing problem, but the funds available from state and federal governments are pitifully inadequate to finance the entire job and the cities appear unable to provide the necessary funds.

As more and more residents move out, increasing numbers and sizes of blighted areas are created, tax delinquency grows, and the cost of city services continues to increase rather than to decrease. The city tax base continues to shrink, and even where real estate bears less than 40% of the burden (as in Kansas City, Mo.) the additional funds collected are barely enough for the cities to provide minimum services.

Eleemosynary institutions are overcrowded and are seriously in need of new buildings and better facilities. Health and welfare programs need expansion and all the cities have a definite need to acquire and develop more playground space conveniently located for each neighborhood. Additional funds are required for community center buildings, and for the provision of better coordinated recreational programs to combat increasing crime, juvenile delinquency, and health problems.

Nevertheless, an ever-increasing percentage of people hope to reside outside the city while comprising its daytime population as workers in business and industrial districts. They wish to enjoy fire protection and other city services for which they do not want to pay and, of course, expect to use the parks, zoos, art galleries, and auditoriums with the same desire to escape the responsibility of paying. Annexation to the city of adjoining areas or an income tax to be paid the city by workers from these outlying areas is fought bitterly.

Schools face increasing difficulties in providing adequate services as the school plant rapidly becomes obsolescent. Many school buildings are outmoded, having been built before the turn of the century on inadequate sites. Population shifts indicate that others should be relocated. Present school locations along major traffic arteries in most instances increase the problems of providing good residential neighborhoods with the best possible living conditions for enjoyment and for creating good citizenship.

Studies of the National Resources Committee indicate the importance of cities in the national economy. Despite the difficulties previously mentioned, cities have many advantages which will continue to attract population. These comments are not to be construed to mean everything is bad in urban areas. Many delightful and charming residential areas are found in new and old sections of cities. Well-developed park and boulevard systems, museums, auditoriums, and cultural centers are available.

Cities are definitely an important part of the national pattern and must be preserved. However, a current diagnosis would indicate that they are suffering from decay, internal pains, poor circulation, congestion, stagnation, and hardening of the arteries, as well as from cancerous growths of slums, which should be cut out and replaced with new tissues lest they spread and absorb unaffected areas.

A more satisfactory urban life must be provided.

APPROACHES TO DECENTRALIZATION CONTROL

By Russell H. Riley,[2] Esq.

The problem of controlling decentralization in cities is a most difficult one. Several steps now appear essential—others may evolve in the future. In general, the character of the problem is similar in all communities, but naturally it is more difficult and more serious in the larger cities. Consequently, the possible solutions are also similar, but the extent of the necessary improvements will vary in cities of different sizes.

Streets, Transit Facilities, and Parking

Since transportation is one of the basic causes of the present problem, it is one of the first fields in which improvements should be considered. Before the advent of the automobile cities were compact, business centers were stabilized, there was little vehicular congestion, and there were no serious depreciating influences on residential property. The automobile enabled the city to expand and precipitated many of the present problems. The automobile cannot be cast aside, however, for people are interested in rapid and convenient movement. Furthermore, the automobile is an essential part of modern economy. It is therefore essential to make such improvements that the motor car can take its proper place in the American community without destroying the desirable and economical form of urban growth. The improvement of streets and transit facilities must be coordinated. Each must perform maximum service in moving people about the city.

Streets.—The majority of existing streets were not planned for use by automobiles. Widenings, new connections, and extensions are essential, but whereas the widening of main thoroughfares will generally serve the traffic needs of the smaller cities (communities comprising not more than 100,000 persons), many widenings are needed in the larger cities and such widenings will not completely solve traffic problems in metropolitan areas. Experience has revealed that any widened street is soon used to capacity, and congestion still exists.

A new type of street—namely, the express highway—must be utilized in larger cities. It is not expected that enough of these can be improved to carry all the automobiles, as the cost would be prohibitive. However, a few should be developed to provide convenient, rapid, and especially safe access between the residential and the business and industrial districts. Only by making the business and industrial centers conveniently accessible to the citizens can their high values be stabilized. Considerable study must be given to the location of these expressways. One major terminus is the central business district; yet, if they are not to encourage further decentralization they should not be extended far beyond the thickly populated part of the urban area. Invariably,

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the greatest delays are encountered in traveling through the older parts of cities, and here the expressway can provide maximum service.

Transit Facilities.—Use of transit facilities declined with the advent of the automobile and, in general, the facilities did not keep pace with other modern improvements. The emergency of World War II clearly revealed the important function that transit can and should still perform. It is obvious, however, that transit vehicles must be modernized and the service so improved that more and more persons will travel by this method. A recent study in a midwestern city (only slightly smaller than Kansas City, Mo.) revealed that 54% of the persons entering the business district during the average day used the transit facilities. The 146,900 that entered by automobile utilized 85,263. The 173,000 that entered via the transit system required only 5,129 vehicles. Whereas the congestion and parking problems could be completely solved by total use of transit vehicles, there is no hope for, and probably no desirability of, entirely replacing the automobile.

The improvement of the transit system will gradually effect a proper adjustment between the use of transit vehicles and automobiles so that a greater number of persons can conveniently enter the downtown business and industrial areas. This is a basic step in stabilizing these centers. The essential improvements will involve new and modern equipment, direct routing on wide streets, and, in the medium size and larger cities, the installation of express service. It is also imperative that transit service be confined to the normally developed urban areas and not serve the sparsely settled outlying sections. This not only benefits the operating company, but it is also an important factor in checking decentralization. Express routes must be designed so that the transit facilities can make maximum use thereof, providing rapid movement through the congested part of the city. Normal transit operations can then be provided on surface streets in the outlying sections of the urban area. This is one of the most important means of expediting the movement of workers and purchasing power to the downtown areas.

Traffic Regulations.—Movement of both automobile and transit vehicles can be greatly expedited by modern traffic regulations. This is especially true of movements within the business district and in other congested parts of the city. The primary function of streets is to accommodate the movement rather than the storing of vehicles. Elimination of parking, especially during rush hours, establishment of one-way streets, and modernization of traffic signals can accomplish outstanding results in moving more vehicles and in moving them rapidly. Experiences in certain cities, such as Philadelphia, Pa., reveal the potential benefits, although it is frequently difficult to secure the support of businessmen and city officials in making such changes. The staggering of working hours is another important way of reducing congestion during peak periods.

Improvements must also be made in the loading and unloading of commodities in the central areas. A single truck can practically destroy one traffic lane for an entire block. Alleys and off-street loading zones are essential in providing a satisfactory solution. In recognition of these facts certain communities have adopted regulations that will gradually improve this condition.

Parking.—To bring more automobiles to the business center will not solve the problem: There must be provisions to accommodate them. Only a small proportion can be parked on the streets, and off-street parking facilities must be provided. As yet there is no uniform agreement as to the location or type of such facilities, or as to who should be financially responsible therefor. From the standpoint of solving the major problem, it is immaterial whether the parking facilities are lots or garages, publicly or privately owned, or where they are located—as long as they are within reasonable distance and can be conveniently used by the traveling public.

Consideration is now being given in several large cities to acquiring parking lots some distance from the shopping center and providing shuttle bus service to the downtown area. This may be a sound solution in larger communities, but it would not be satisfactory in the smaller areas. Each city must study and develop the type of parking that seems best suited for its particular requirements. The important factor is that adequate off-street parking facilities be provided.

Zoning and Rehabilitation

Another important reason for decentralization is the intermingling of property uses and the undesirable living conditions in the older sections of cities. Not only was zoning begun too late to prevent this condition in larger cities; but also, all too frequently, the present zoning regulations permit further depreciating influences in these older areas. The majority of zoning ordinances are not in scale with current needs. There is far too much commercial and industrial zoning in sections that are and should continue to be, primarily residential.

Zoning can be one of the most important factors in checking decentralization, but it must be a positive rather than a negative force. It must completly protect the good and fair residential districts so that they will not be injured by inappropriate uses, and it must also protect the older blighted districts that can only be logically used for residential purposes. It must be a positive guide for the future organization of the city and must give assurance that the older areas can be rehabilitated or redeveloped for housing purposes without recurrence of the same damaging influences that caused their depreciation.

Rehabilitation or redevelopment is a comparatively new field in cities of the United States. There are few actual examples indicating how blighted areas can be cleared and rebuilt for residential purposes. However, these few examples reveal great potentialities, and it is encouraging to note that many states have recently enacted laws facilitating redevelopment, especially by private capital. Comprehensive studies are also being made for redevelopment projects in many cities.

It is unthinkable that a country as young and as powerful as the United States would continue to stand the economic waste and the undesirable living conditions that are found in extensive parts of the larger cities. Both private and public capital must adopt widespread programs for the gradual rebuilding of these blighted areas in a manner that will prevent a recurrence. This is a large and difficult task, yet the few examples indicate that it is not impossible.

It is sound business to remove an uneconomical section of the urban area and to replace it with an attractive, livable, and revenue-producing development. The rebuilding of blighted areas is a fundamental step in checking decentralization.

Other Physical Improvements

Schools, parks, sewers, public buildings, and other public improvements are seriously needed in practically all communities. Cities grew so rapidly that these facilities seldom kept pace with the growth. Furthermore, new standards are frequently being evolved so that extensive improvements are needed to meet modern requirements. These physical facilities are not mere luxuries—they are minimum essentials. Although frequently not found in the newly developing outlying sections, the population soon presents a strong demand for them. Much can be done to stabilize residential growth if adequate and modern systems of these improvements are provided in the existing city. There should also be a definite policy that no additional facilities will be developed until such time as the area served by the existing facilities has been completely absorbed by urban growth. The location and character of improving essential public facilities can thus be a very important factor in checking outlying growth. It also enables the city to develop a long-range public works plan and to budget its expenditures accordingly. The cost of providing such facilities is admittedly large, yet it is small in comparison with the cost of decentralization or with the cost of continuously extending improvements over a never-ending area.

Guidance of New Growth

Irrespective of the effectiveness of the methods previously outlined, there will always be some outlying growth and development—it is the inherent tendency of American people to move to new locations. While an excessive amount of decentralization must and can be controlled, it is equally important that such outlying growth as does occur conform to modern desirable standards. It is particularly important that the new growth be compact rather than widely scattered. It is the scattered development in which the cost of improvements and services is abnormally high that cannot be economically justified. On the other hand, the development of complete, compact communities containing a population that can be economically provided with public improvements and services does not present serious problems of decentralization. Such communities even have an important advantage in that they can be surrounded by rural development, which should be an important means of protecting living conditions and property values.

County zoning regulations and good subdivision control are the most effective methods of bringing about a compact and economical pattern of outlying growth. Subdivision regulations that require the installation of all necessary improvements before lots are sold can be particularly effective in checking unwarranted outlying growth and in securing a desirable standard of improvements. Likewise, county zoning regulations, based on careful studies of the population that should eventually be located in the outlying areas, and which provide for the compact grouping of this growth, are valuable aids. Under

such zoning the remainder of the unincorporated area can be restricted to rural uses and large lots, which are its most logical use.

Distribution of Cost and Taxes

A reason frequently advanced for persons leaving the older city for the outlying areas is to escape the higher taxes. This is understandable. It is seldom realized over a long period, however, for eventually the cost of providing improvements and services in the outlying areas to the standards to which the people were accustomed frequently involves taxes higher than those paid in the large city. It is also fundamentally inequitable for persons living beyond the corporate limits of the cities to utilize and benefit from the facilities of the city without contributing to their improvement and maintenance. Such facilities include major streets, parks, libraries, and other similar conveniences. There appears to be logical justification for a metropolitan form of government comprising all the urban area, and which would tax this area for such improvements as streets, libraries, parks and other facilities that are used by the entire population. This would not only insure the center city of receiving some assistance for the improvements used by all persons, but it would probably result in a better standard of such facilities in the outlying areas. At the same time, it would permit the continuance of smaller local governments, which would provide improvements and services used primarily by the citizens in that particular community.

Public Understanding

In the final analysis it is public understanding that will, to a large extent, determine whether decentralization will continue as at present or whether it will be effectively controlled. The people must know how they are affected by decentralization, they must understand the cost of the blighted areas—the cost that will result in installing a satisfactory standard of improvements and services in outlying areas—and the hazards and inconveniences that will result from inadequate improvements. They must realize that no matter where they live, they have a definite personal interest in the entire metropolitan area and that each one will be affected by what happens in any part of this large area. There must be widespread dissemination of data and facts regarding the problem, the needs, and the possible solutions.

Of particular importance is the fact that there must be aggressive leadership. Only by widespread understanding on the part of the citizens and by aggressive leadership can the essential results be obtained. Admittedly, the problem is extremely difficult and serious, yet the potential benefits are so great that every likely solution must be examined and every possible effort must be exerted.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

EXPERIENCES WITH PREDETERMINING PILE LENGTHS

By William W. Moore,[1] M. ASCE

Synopsis

In the design of pile foundations there are two problems for which no adequate solutions have been found in common engineering practice. One of these problems is to estimate the required lengths and probable behavior of different types of piling when it is not feasible or economical to perform full-scale driving and loading tests previous to preparation of complete construction plans. The other is to evaluate the significance of various factors that affect the behavior of piles both during driving and under test loads. The methods described for predetermining pile lengths and bearing capacities are not offered as a final answer to all problems in the design of pile foundations, but rather are intended to complement other methods of determining the probable safety of such foundations. These methods can be used for evaluating the bearing capacity of single piles—however, determination of the probable settlements and safe loadings for large groups of piles requires other and additional analyses. Incompleteness of knowledge and imperfectness of materials impose such wide limits of accuracy on pile foundation analyses that most conclusions must, to some degree, be empirically based upon observations and performance records. Typical foundation designs employing various types of piling are presented to illustrate the methods described.

Introduction

There have been attempts by some investigators to evaluate the supporting capacity of piling in terms of the skin friction which the soil would offer on the embedded surface area of the pile. All these schemes have followed the line of observing the results of field loading tests on piles and dividing the tested capacity by the gross surface area of the pile, and then trying to apply the information to some other site where the soils, by visual examination, appeared similar.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by April 1, 1948.

[1] Partner, Dames & Moore, San Francisco, Calif.

The methods outlined herein result from an attempt to arrive at a more rational design for pile foundations which would be more consistent with the improvements in structural analysis than are the older and more or less generally accepted dynamic pile driving formulas. This approach must be combined with good judgment and the lessons of experience in practical foundation construction, and, when so used, may help to avoid some pitfalls and failures.

From experience records a few typical cases have been selected to illustrate the design of foundations employing wood piling, cast-in-place concrete piling, pre-cast concrete piling, and steel H-piling.

Wood Piling

Design of a wood pile foundation is typified by one for support of the Submarine Spare Parts Building, a wood frame industrial warehouse situated on reclaimed marshlands within the United States Navy Yard, Mare Island, California. Explorations were made with wash boring equipment, and undisturbed core samples of the soils were extracted by means of a driven tubular sampler for laboratory inspection and testing. Typical soil conditions underlying this construction area are given in Table 1. Physical properties of the soils as determined by laboratory tests using a ring-type double-shear apparatus for determining shear and friction values are given in Table 2. The tests were generally

TABLE 1.—Soils at Site of Submarine Spare Parts Building, United States Navy Yard, Mare Island, California

Depth below surface (ft)	Type	Color	Remarks
0 to 7..	Sandy loam, rocks	Brown	Fill
7 to 15..	Silty loam, decayed vegetation	Black	Soft bay mud
15 to 26..	Silty loam	Dark gray	
26 to 40..	Peaty clay	Brown	
40 to 47..	Peaty clay	Dark gray	
47 to 54..	Clay	Blue-green	Firm clayey soils
54 to 65..	Clay and shells	Light brown	
65 to 70..	Clay	Dark gray	
70 to 80..	Clay	Blue-green	
80 to 90..	Clay	Light brown	

TABLE 2.—Physical Properties of Soils in Table 1

Depth[a] below surface (ft)	Surcharge pressure (lb per sq ft)	Moisture content (%)[b]	Dry density (lb per cu ft)	Shearing strength (lb per sq ft)	Frictional resistance on wood (lb per sq ft)	Yield point bearing (lb per sq ft)
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1..........	100	17.0	106.9	6,400+	....	20,000+[c]
8..........	950	94.0	48.5	160	160	500
20..........	1,450	107.6	42.2	240	....	750
30..........	1,950	131.0	35.2	360	320	1,125
40..........	2,550	69.1	56.6	400	....	1,250
50..........	500	24.8	100.5	950	....	3,000
50..........	3,300	24.9	102.1	1,100	1,100	3,500
60..........	3,900	42.5	79.2	1,430	1,110	4,500
70..........	4,500	44.5	75.5	1,190	1,030	3,750
80..........	4,100	41.1	77.7	1,590	....	5,000
89..........	4,750	36.3	82.7	2.380	1,590	7,500

[a] Point at which undisturbed sample was taken. [b] Percentage of dry weight. [c] Rock fragments in shear planes; not a representative test.

made with a confining surcharge pressure that approximately equaled the weight of the field overburden after compensation for the buoyant effect of the ground water. Col. 5, Table 2, indicates the yield point shearing strengths. For the purpose of assisting a general appreciation of the qualities of the foundation soils, the equivalent yield point bearing values[2] are given in Col. 7. The contrast between the soft bay mud and the firmer clayey soils is evident by the differences in strengths, moisture contents, and densities. The frictional values of the soils in contact with wood (Col. 6), representing wood piling, also indicate the marked difference between the soft mud and the firm clays.

A summary of the results of the shear and friction tests performed on the soil samples extracted from the two borings on this site is shown in Fig. 1. This summary was used to select representative values of the shearing strengths of the soft bay deposits and the firmer clayey soils (the two principal soil types) and representative values of the frictional resistances offered by these materials to wood piling.

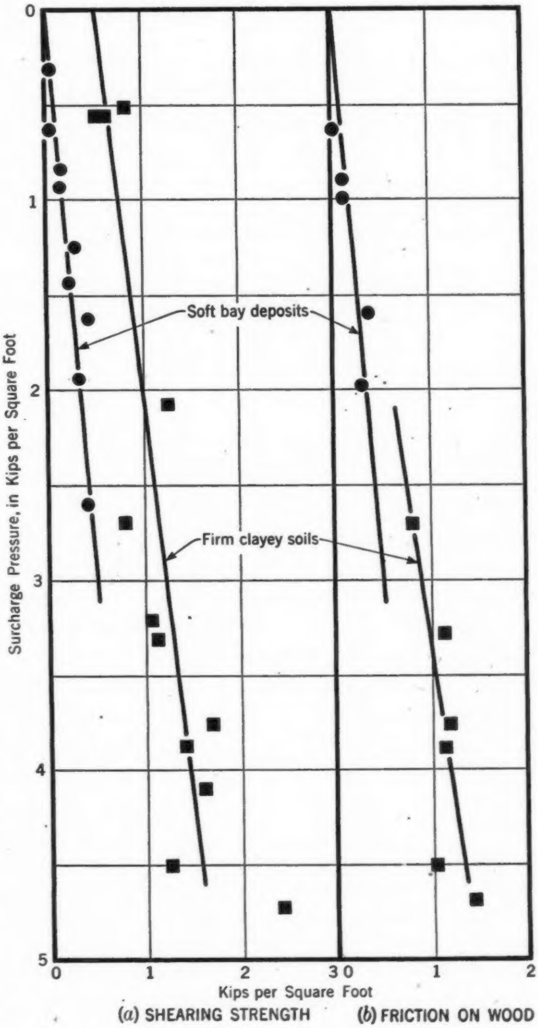

Fig. 1.—Summary of Soil Test Data from Two Borings at Mare Island (California) Site

The nature of the forces considered to be exerted on the pile by the soil are shown by vectors in Fig. 2. These forces consist of horizontal compressive forces perpendicular to the surface of the pile, and vertical shearing (or frictional) forces parallel to the axis of the pile. It is assumed that the force perpendicular to the surface of the pile at any point is a function of the weight of the soil overburden above

[2] "Practical Shear Tests for Foundation Design," by Trent R. Dames, *Civil Engineering*, December, 1940, pp. 784–785.

that point and may be greater than the weight of overburden by a factor which depends upon the shearing resistance of the soil—that is, the driving of the pile into the ground will displace the soil so that an increased lateral pressure (similar to "passive resistance") is created on the surface of the pile. The actual value of this lateral pressure is believed to be dependent not only on the shearing strength of the soil, but also, in some unknown degree, on the compressibility of the soil and the volume of soil displaced by the pile. Thus,

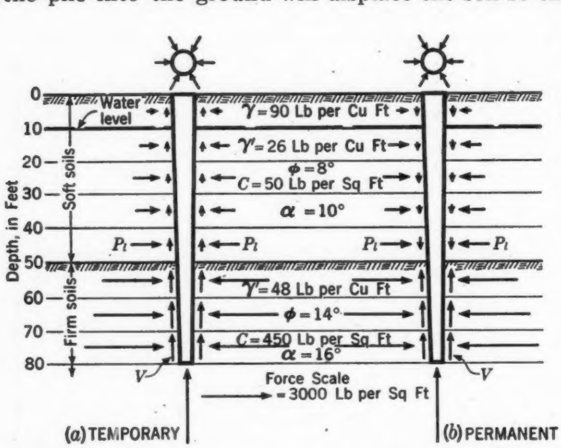

Fig. 2.—Diagrams of Lateral and Supporting Forces on Wood Piling Having 8-In. Tip and Tapering 1 In. in 10 Ft

$$P_l = K \sum \gamma \, h + f(S) \quad \text{(1)}$$

in which P_l is the lateral pressure; γ is the unit weight of the soil; h is the depth to the point being considered; and $f(S)$ is a function of the shearing strength of the soil. The increased pressure created by displacement of the soils during driving is believed to be, generally, on the order of from three to five times the shearing strength, although for jetted piles or piles of small displacement there may be no appreciable increase in the lateral pressure. The potential supporting forces parallel to the axis of the pile are assumed to be limited either by the friction which would be developed by the estimated lateral pressures, or by the shearing strengths of the soils surrounding the pile, whichever is less. Thus,

$$V \leqq P_l \tan \alpha \quad \text{(2)}$$

or

$$V \leqq S \quad \text{(3)}$$

in which V is the supporting force parallel to the axis of the pile; α is the angle of friction; and S is the shearing strength of the soil. The method of evaluating the total capacity of the pile to support external loads consists of an arithmetic summation of the potential shearing or frictional forces in each of the soil strata as shown by the computation in Table 3. In Table 3, the values in Cols. 3, 4, 5, and 6 are calculated from

$$S = C + P \tan \phi \quad \text{(4)}$$

$$F = P \tan \alpha \quad \text{(5)}$$

$$F_m = F \frac{\pi S + P}{P} \quad \text{(6)}$$

$$B = \pi S \left(\frac{\pi d^2}{4} \right) \quad \text{(7)}$$

respectively, in which C is the cohesion of the soil; P is the surcharge pressure; ϕ is the angle of internal friction; F is the average friction; F_m is the friction modified; B is the end bearing of the pile; and d is the diameter of the pile at its tip.

TABLE 3.—COMPUTATION OF TEMPORARY AND PERMANENT CAPACITIES OF WOOD PILE SHOWN IN FIG. 2

Depth below surface (ft)	Surcharge pressure, P (lb per sq ft)	Average shear, S (lb per sq ft)[a]	Average friction, F (lb per sq ft)	Modified friction, F_m (lb per sq ft)	End bearing, B (lb)	SUPPORT OR DRAG (IN KIPS) FOR PENETRATION INTO FIRM CLAY (FT)				
						0	10	20	30	40
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(a) Temporary Support or Potential Drag										
0										
	450	113	79	141		3.70	3.99	4.29	4.58	4.88
10	900									
	1,030	195	182	290		5.87	6.38	6.89	7.40	7.91
20	1,160									
	1,290	231	227	354		6.35	6.95	7.56	8.16	8.77
30	1,420									
	1,550	268	273	421		6.67	7.38	8.07	8.77	9.47
40	1,680									
	1,810	304	319	487		6.76	7.56	8.37	9.15	9.95
Total						29.35	32.26	35.18	38.06	40.98
(b) Permanent Support										
50	1,940	933			1,026	1.03				
	2,180	993	625	1,518		30.38	22.10	24.70	27.30	29.9
60	2,420	1,053			2,662		2.66			
	2,660	1,113	762	1,763			57.02	24.77	27.68	30.60
70	2,900	1,173			3,190			3.19		
	3,140	1,232	900	2,009				87.84	27.41	30.64
80	3,380	1,293			3,718				3.72	
	3,620	1,352	1,038	2,255					124.17	30.09
90	3,860	1,412			4,246					
										162.21
Temporary pile capacities, in tons						15.19	28.51	43.92	62.08	81.10
Permanent pile capacities, in tons						−14.16	−3.75	8.74	24.02	40.12

[a] Controlling values.

The results of the pile capacity analyses may be shown graphically, as in Fig. 3, from which the desired pile lengths may be taken directly. The pile capacities are considered in two parts. The first part of the analysis considers the condition after the piling is driven and before sufficient time has elapsed to permit subsidence of the fill and soft bay mud. Under this condition, all the frictional forces would tend to support the piling; capacities for this condition are indicated by curve (*a*), Fig. 3. The second part of the analysis considers the condition when sufficient subsidence of the fill and compression of the soft bay mud has occurred to develop the frictional forces which will tend to drag

the pile downward. Under this condition, the calculations indicate that a penetration of approximately 15 ft into the firm clayey soils would be required to resist the forces which would be applied by the subsiding fill and bay mud. Any additional penetration into the firm clayey soils would provide a potential resistance which might be utilized to support the structural loads—for example, a penetration of 34 ft into the firm clayey soils would provide adequate capacity to support a 30-ton total design load. These capacities are indicated by curve (*b*). In connection with these curves it should be noted that, for the particular structure considered, it was decided that the recommended maximum dead load should not exceed two thirds of the permanent pile capacity.

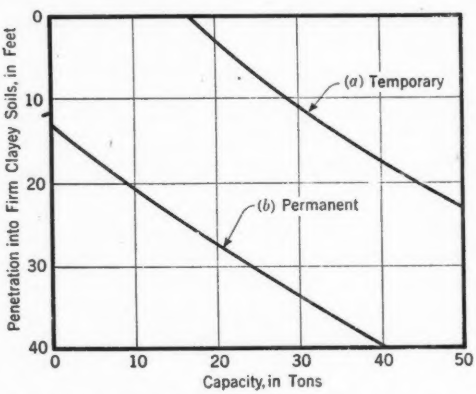

Fig. 3.—Computed Capacity of Single Wood Piles, or Each Pile of Small Groups, at Mare Island (California) Site

Some variation in the surface elevations of the firm clayey soils was disclosed over the site. Data obtained by borings and driven test rods enabled construction of surface contours of the firm clayey soils. Use of these contours in conjunction with the calculated penetrations into the firm soils required to support the proposed loads enabled selection of the lengths of piling required for the various parts of the proposed structure.

After calculating the pile capacities, based on the field explorations and laboratory tests, it was desired that some full-scale confirmation of the computed capacities be obtained. Therefore, field loading tests were conducted on wood piles driven on a site adjacent to that for which the explorations already described were made. The soil conditions were very similar to those illustrated, but differed in the thickness of fill and depth of soft bay mud. The pile loading test setup is illustrated in Fig. 4. The 67-ft-long test pile was between two anchor piles, and a reaction girder was framed between the two anchor piles so that load could be applied to the test pile by gear-driven screw jacks, using calibrated beam gages to measure the applied loads. Settlements of the test pile were recorded by micrometer dial gages reading to 0.001 in., the dial gages being supported on a reference beam independent of the loaded elements. The dial gage, which indicates settlements of the test pile, may be noted in front of the left jack in Fig. 4.

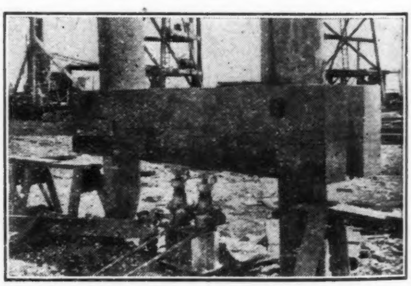

Fig. 4.—Setup for Wood Pile Loading Test

Loads were applied to the test pile in increments, each load being maintained constant until settlement observations indicated that the rate of subsidence had substantially ceased. Each load was usually held until movement of less than 0.001 in. was observed in a 15-min. period. At certain loadings, the load on the test pile was removed and then reapplied in order to observe the effect of repeated loading in producing progressive settlements. Results of the loading test are shown in Fig. 5. Since the loading test was conducted

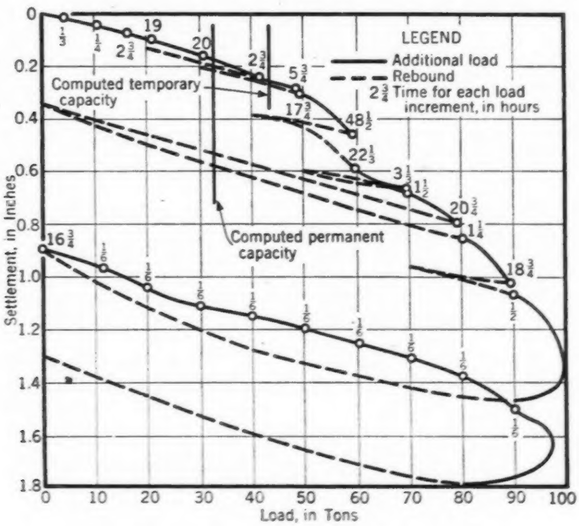

FIG. 5.—SETTLEMENT OF WOOD TEST PILE UNDER LOAD.

only a few days after the pile was driven, it was expected that the frictional forces of all the soils surrounding the pile, including the fill and soft bay mud, would assist in supporting the pile during the test. Therefore, it was expected that the test would be in substantial agreement with the computed temporary pile capacity. Computed pile capacities were determined by the method previously described, adjustments being made to conform with the depths of fill and bay mud at the location of the pile test. The relationship between the computed temporary pile capacity and the settlement of the pile under load is shown in Fig. 5. Yielding of the pile under load occurred in the vicinity of 50 tons as shown by a rather definite break in the slope of the curve and the increasing permanent settlements observed under repeated loadings. The computed permanent pile capacity, with due allowance for the frictional forces which would be imposed by subsidence of the fill and compression of the soft bay mud, is also indicated in Fig. 5. Loads on the test pile were increased until ultimate failure was observed under a load of approximately 100 tons.

Since the results of the loading test were in substantial agreement with the computed pile capacity, the lengths of piling to be driven for the proposed building were selected as previously outlined, and the final lengths of the piling driven agreed quite closely with the predetermined lengths, the extreme

variations being from 4 ft shorter to 2 ft longer than those computed. Variations between driven lengths and computed lengths were due to variations in lengths of available stock; except for a few cases where the piles were driven to refusal above the computed tip elevations.

Although the driving resistances, as computed by the *Engineering News* formula, varied from approximately 9 tons to 35 tons because of such factors as variations in straightness, grain of the wood, squareness of the blow on the pile, and variations in the soils over the site, all the piles were capable of supporting the 30-ton total design load. Since completion of the building in the fall of 1942, observations indicate that the floor, which was not supported on piling, has settled slightly more than 1 ft. Although no continuous instrumental observations of the elevations of the pile caps are available since erection of the building, such observations as have been made indicate that if there has been any settlement of the piling it is on the order of small fractions of an inch.

A near-by structure, of a very heavy and rigid type of reinforced concrete construction, is also supported on piling driven in substantially the same soils as those described, and the lengths of piling driven were determined in the same fashion. Observations of this building also indicate that no measurable settlements have occurred—in this type of structure the occurrence of any appreciable differential settlements would certainly be evidenced by severe cracking of the walls and floors.

Cast-in-Place Concrete Piling

Use of the method in a design involving cast-in-place concrete piles is illustrated by a foundation for the support of the fluid catalytic cracking unit in the Avon Refinery of the Tide Water Associated Oil Company at Associated, Calif. Typical soil conditions are given in Table 4, and the physical properties of the soils in Table 5. It will be noticed that the soils were moderately firm to firm clays interspersed with strata of fine sand. Design values of the principal soil properties selected for use in calculating capacities of steel-shelled cast-in-place concrete piling and wood piling for various lengths of embedment in these soils are given in Fig. 6. In selecting representative design values for the shearing strengths of the soils, rather conservative interpretations of the data have been made.

TABLE 4.—Soils at Oil Refinery Site at Associated, Calif.

Depth below surface (ft)	Type	Color
0 to 4½....	Clay	Dark brown
4½ to 11....	Clay	Light brown
11 to 18....	Fine sand	Brown
18 to 30....	Clay	Light brown
30 to 40....	Fine sand	Brown

Because of the lack of continuity of the sand strata beneath the proposed structure, the analyses were made so that the pile capacities would be adequate to support the design loads where the soils were predominately of clayey nature; and, where sand strata of considerable extent were encountered, an additional factor of safety would be realized. Results of the analyses of pile capacities are shown in Fig. 7.

TABLE 5.—PHYSICAL PROPERTIES OF SOILS IN TABLE 4

Depth[a] below surface (ft)	Surcharge pressure (lb per sq ft)	Moisture content (%)[b]	Dry density (lb per cu ft)	Shearing strength (lb per sq ft)	Frictional resistance on wood (lb per sq ft)	Friction on steel (lb per sq ft)	Yield point bearing (lb per sq ft)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1.7	200	29.5	91.3	1,590	40	...	5,000
5	600	21.0	107.2	950	...	160	3,000
9	1,100	29.8	91.9	640	160	...	2,000
12	1,400	24.7	101.0	1,000	...	440	3,130
19.5	1,850	21.1	105.9	1,750	480	...	5,500
25	2,200	20.6	107.5	1,750	...	790	5,500
32	2,700	21.0	106.2	2,070	790	...	6,500
39.5	3,200	20.1	108.2	2,540	...	1,030	8,000

[a] Point at which undisturbed sample was taken. [b] Percentage of dry weight.

It was expected that considerable variation in the driving resistance of the piles would be observed because of discontinuity of the sand strata. However, as indicated by the pile capacity analysis, an adequate capacity to support the

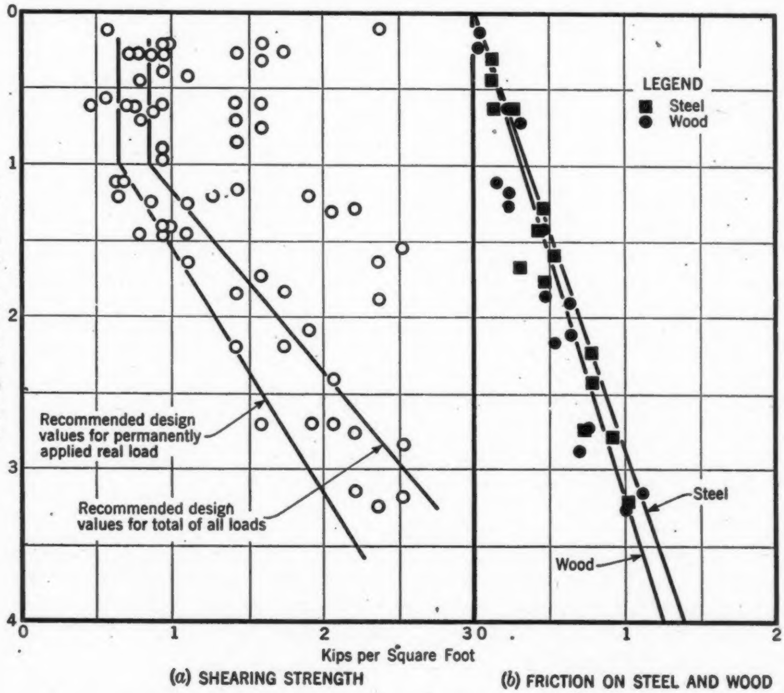

FIG. 6.—SUMMARY OF SOIL TEST DATA AT REFINERY SITE

design load of 40 tons per pile was assured at all locations on the site by cast-in-place concrete piles penetrating to approximately 30 ft below the ground surface, with the pile cap placed at a 5-ft depth. Experience during the driving

of the piles indicated that, where the lower layer of sand was not encountered, use of the *Engineering News* formula would have required an additional 10 ft or more of penetration to support the design loads. According to the *Engineering News* formula, bearing capacities generally on the order of 20 tons and higher were developed. The cost of unnecessary increased lengths of the piling of course would have been a considerable item.

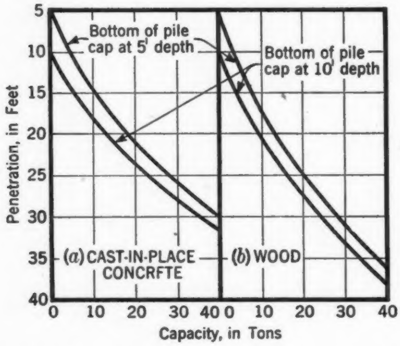

Fig. 7.—Computed Capacity of Single Piles, or Each Pile of Small Groups, at Oil Refinery Site

The fluid catalytic cracking unit, completed late in 1944, imposes a dead load of approximately 8,700 tons, on 352 step-tapered, cast-in-place, concrete piles 25 ft long, or slightly more than 24 tons of dead load per pile.

Observations of the settlement of the structure since its erection have been recorded by the engineering forces of the owner. Plottings of these readings indicate some apparently erratic readings, which are believed to represent the approximate accuracy of the observations —that is, it is impracticable to measure, with an ordinary engineer's level, settlements of the order of those that have taken place. In fact, the greatest settlement thus measured during the first year after completion of the unit was 0.012 ft.

Pre-Cast Concrete Piling

Soil conditions into which the pre-cast concrete piling to support a reinforced concrete pier at a shipyard in Richmond, Calif., was driven are given in Table 6. The physical properties of these soils are given in Table 7. All the soils below the dredged depth, approximately 32 ft below mean lower low water, are moderately firm clays. The summary of shear test results (Fig. 8(*a*)) indicates that there is no dependable increase in the shearing strength of the clay with depth. The design values of shearing strength and frictional resistance indicated in Figs. 8(*a*) and 8(*b*) were used as previously described to compute the support offered to pre-cast concrete piling embedded in these soils. Results of these pile capacity analyses are shown in Fig. 9.

TABLE 6.—Soils at Shipyard Site at Richmond, Calif.

Depth below MLLW[a] (ft)	Type	Color
0 to 32.5	Water	....
32.5 to 39	Clay	Greenish gray
39 to 53.5	Clay	Brown
53.5 to 62.5	Clay	Gray
62.5 to 87.5	Clay	Dark gray

[a] Mean lower low water.

A load test was made on a pile typical of those to be used in the structure. Elements of the loading test setup were similar to the setup previously described for the wood pile test except that at the test location, which was in the bay, it was impracticable to support direct recording deflection instruments to record the settlements, and consequently settlement measurements were made

TABLE 7.—PHYSICAL PROPERTIES OF SOILS IN TABLE 6

Depth below MLLW[a] (ft)	Surcharge pressure (lb per sq ft)	Moisture content (%)[b]	Dry density (lb per sq cu ft)	Shearing strength (lb per sq ft)	Frictional Resistance (Lb per Sq Ft)			Yield point bearing (lb per sq ft)
					On wood	On steel	On concrete	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
45.5......	850	28.3	97.0	1,110	400	...	...	3,500
54.......	1,300	35.2	86.8	960	...	480	...	3,000
58.5......	1,500	68.7	60.1	640	...	...	480	2,000
71.5......	2,050	46.5	74.7	1,430	640	...	...	4,500
80.......	2,450	33.4	89.0	1,910	...	880	...	6,000
86.5......	2,800	31.6	91.5	1,110	...	...	1,040	3,500

[a] Depth below mean lower low water at which undisturbed sample was taken. [b] Percentage of dry weight.

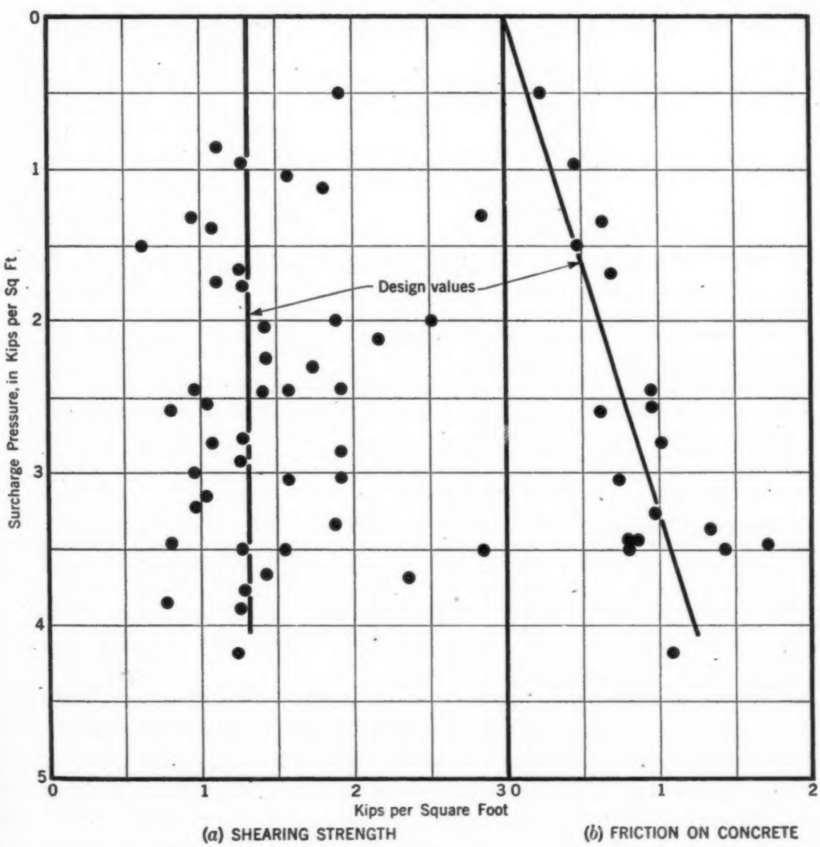

FIG. 8.—SUMMARY OF SOIL TEST DATA AT SHIPYARD SITE

with an engineer's level securely mounted on a three-pile dolphin having no structural connection with the loaded elements. Considerable difference was observed between the driving resistances for the test pile and those for the anchor piles, but these are attributed to differences in the shapes of the piles rather than to any important difference in the strengths of the soils penetrated. Settlements of the test pile under load are indicated in Fig. 10, in which is also shown the relationship of the yielding of the pile under load to the computed capacity. The load-uplift relations for the anchor piles are shown in Fig. 11.

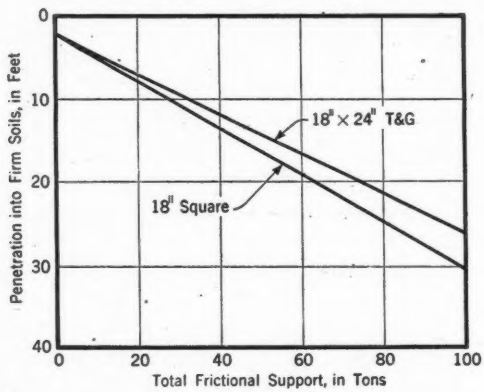

FIG. 9.—COMPUTED PERMANENT CAPACITY OF PRE-CAST CONCRETE PILES AT SHIPYARD SITE

In addition to determining capacities of the individual piles, it was desired to evaluate the probable settlement of the structure that would result from compression of the soils supporting the entire pile foundation. Stress conditions in the principal supporting soils are shown in Fig. 12. The analysis was made assuming that the settlement would be the same as though the structure were supported on a spread foundation at the elevation of the pile tips. The width of the equivalent spread foundation was assumed equal to the width of the pile group plus an additional width equal to the depth of penetration of the piles into the firm soils. Consolidation tests on the firm clays were used in conjunction with the indicated stresses to evaluate the order of magnitude of settlement which would be expected during the life of this structure. The magnitude of these settlements is indicated by Fig. 13. This range of expected settlements is intended to represent the order of accuracy of the analysis—that is, the actual settlement should fall within the range shown if the design loads were permanently applied.

In driving the permanent piles for support of the pier, it was specified that they be driven to the tip elevations indicated by the analyses, provided that this depth could practicably be reached without damaging the piles during driving and that the final driving resistance did not fall below nine blows per foot with an OR Vulcan hammer. This procedure was adopted to disclose any unexpectedly soft soils. The pre-cast concrete piles were of two types: 18-in.-square and 18-in. by 24-in. tongue and groove sheet piling. Except for a few piles in locations subjected to special loading conditions in which the

piles were driven to elevations as deep as −72 ft, the 18-in.-square piles were generally driven to predetermined tip elevations of from −62 ft to −64 ft, or approximately 32 ft into the moderately firm clays. The *Engineering News* formula indicated safe loads ranging from 27 tons to 65 tons, whereas the computed capacity for these piles was approximately 100 tons. The 18-in. by 24-in. tongue and groove sheet piles were generally driven to a tip elevation of −57 ft and, being of slightly larger size, were computed to develop approximately the same ultimate bearing capacity by skin friction as the 18-in.-square piles. The driving resistances of these piles, however, varied from 40 tons to 105 tons as indicated by the *Engineering News* formula. All the piles were driven with the same hammer, delivering a blow of 24,450 ft-lb with an 8,000-lb ram. Very few piles were cut off at all. Such cutoffs as were made generally resulted from the use of piles cast longer than required for a given location. The dead load imposed by the structure on the piles varies from 18 tons to 28 tons, and the maximum dead and live loads from about 60 tons to 66 tons, all of which was regarded as potential real load.

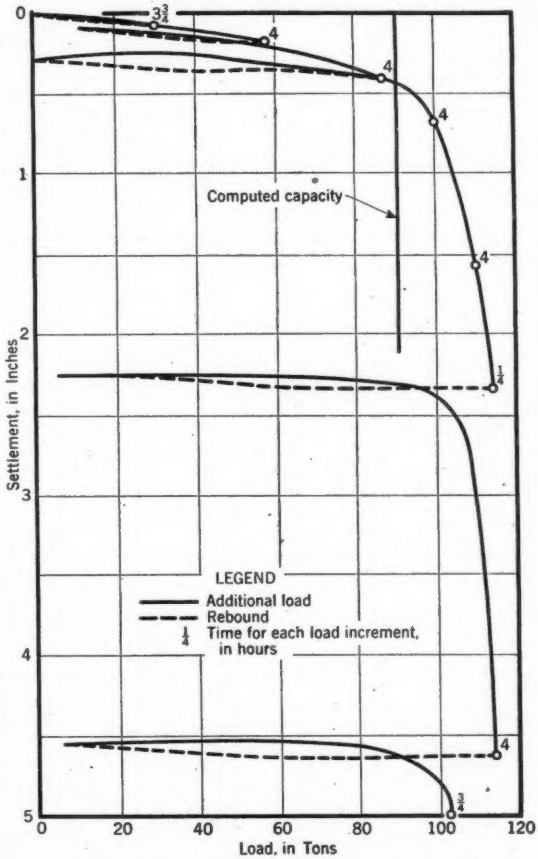

FIG. 10.—SETTLEMENT UNDER LOAD OF 18-IN. BY 24-IN. TONGUE AND GROOVE PRE-CAST CONCRETE TEST PILE AT SHIPYARD SITE

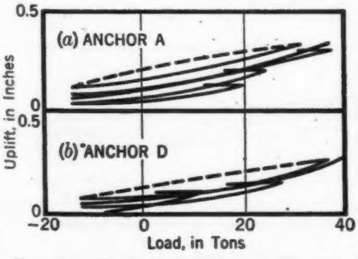

FIG. 11.—LOAD-UPLIFT RELATION FOR ANCHOR PILES A AND D, SHIPYARD SITE

During the interval since completion of the pier, it has been subjected to crane loads and deck loads in some cases exceeding those in the design. At one time a deck load of about 1,300 lb per sq ft was applied for a period of several days. Level readings on the crane rails indicate deflections varying from an indicated uplift of 0.02 ft to an indicated settlement of 0.02 ft at different points along the pier.

Another example of the capacity of pre-cast concrete piling which may be of interest is that of a loading test made for the California Point Ammunition Loading Project which the United States Engineer Office was contemplating at one time. The data from one of these pile loading tests are shown in Figs. 14 and 15. The test pile was placed into soft bay mud, the surface of which was approximately 30 ft below mean lower low water. The driving resistance of the pile was practically zero—that is, the pile was placed to its final tip elevation without a blow from the hammer, having penetrated more than 50 ft under its own weight, and another 10 ft under the additional weight of the hammer. When test loads were applied thirteen days after the pile had been driven, the data indicated by Figs. 14 and 15 were obtained. Calculations of the pile capacity based on shear and friction tests on core samples of the soil indicated that the pile should be capable of supporting loads of about 42 tons. As may be observed from Fig. 14, the load actually carried before yielding is in much closer agreement with the computed capacity than it is with the capacity indicated by a dynamic formula. This is, of course, one of the most extreme cases that could be selected, but nevertheless it is a very practical one.

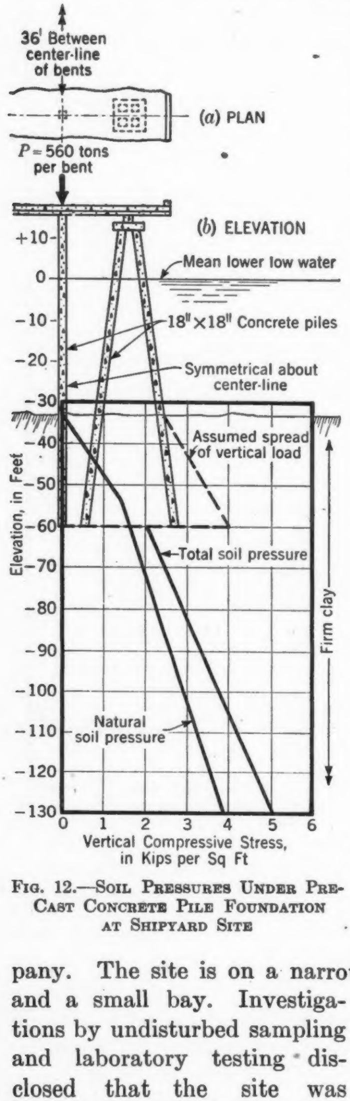

FIG. 12.—SOIL PRESSURES UNDER PRE-CAST CONCRETE PILE FOUNDATION AT SHIPYARD SITE

STEEL H-PILING

The last case selected to illustrate this method of predetermining pile lengths is that of the new Ras Tanura Refinery in Saudi Arabia, for the Arabian American Oil Company. The site is on a narrow sandy peninsula between the Persian Gulf and a small bay. Investigations by undisturbed sampling and laboratory testing disclosed that the site was covered by various thicknesses of sands and sandy loam soils, sometimes interspersed with very soft compressible strata. At depths ranging from 5 ft to as much as 20 ft, there was found an extensive bed of rather weak sandstone, which comprised the

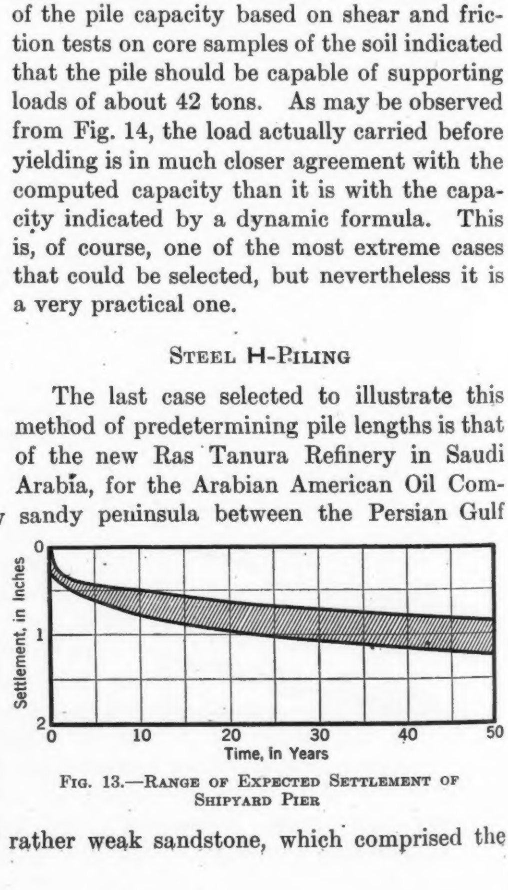

FIG. 13.—RANGE OF EXPECTED SETTLEMENT OF SHIPYARD PIER

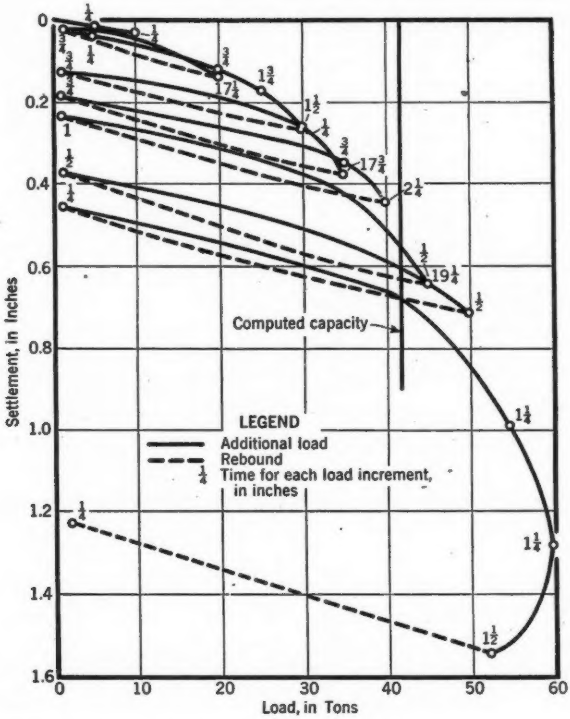

FIG. 14.—SETTLEMENT UNDER LOAD OF PRE-CAST CONCRETE TEST PILE AT CALIFORNIA POINT SITE

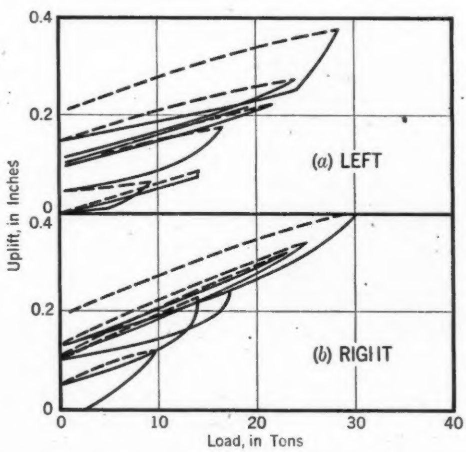

FIG. 15.—LOAD-UPLIFT RELATION FOR ANCHOR PILES, CALIFORNIA POINT SITE

principal supporting stratum for the refinery foundations. A representation of the typical sequence of soils from the surface down to the sandstone is presented in Fig. 16, which also gives the results of computations of bearing capacities of steel H-piling having various penetrations into the soils on this site.

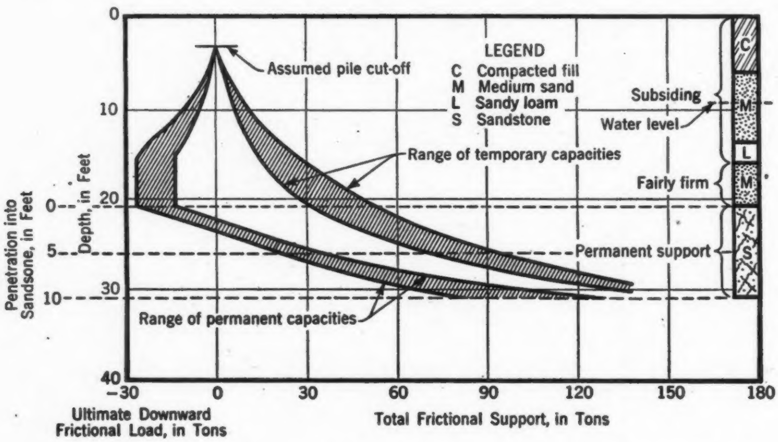

FIG. 16.—COMPUTED CAPACITY OF STEEL H-PILES AT ARABIAN OIL REFINERY SITE

A loading test was made early in the construction program within a few hundred feet of the area covered in this analysis. The test pile was a 12H53 shape driven with a Vulcan No. 1 hammer to a depth of 17 ft below the ground surface, which resulted in approximately 9 ft of penetration into the sandstone. The dynamic driving resistance reached approximately 0.5 in. per blow—thirteen blows for the last 6 in. Under the application of test loads up to 65 tons, a total settlement of about 0.08 in. was observed without any indication of yield being developed. Unfortunately, the reaction frame for this test was not strong enough to permit carrying the test to ultimate failure, but the results did add considerably to the knowledge of the bearing capacities of piling for this project.

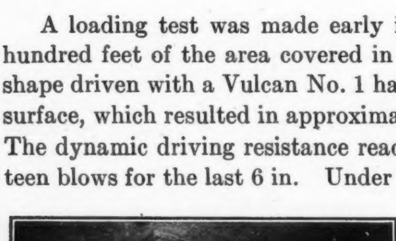

FIG. 17.—SETTLEMENT AWAY FROM STRUCTURE OF WOOD PILE DRIVEN THROUGH ROCK FILL AND SOFT MUD INTO FIRMER SOILS

After erection of some of the heavier refinery vessels, the units were filled with water for testing. The water load, although applied for a period of only a few hours, imposed more severe forces on the foundations than would be expected at any other time during the life of the structures. Level readings taken during the application of this water load on one of the heaviest units showed maximum settlements on the order of $\frac{1}{4}$ in.

Conclusion

The methods of analysis employed for predetermining pile lengths are in a formative stage, and it is hoped they will be improved as knowledge increases. This approach to pile foundation design should be of considerable assistance in avoiding situations such as those shown in Fig. 17, which represents a condition disclosed while investigating the settlement of a waterfront structure where piling had been driven through rocky fill material into firmer soils. Driving resistances were ample according to dynamic pile driving formulas; but the force imposed on the pile by subsidence of the fill and compression of the soft bay deposits was too great, and the pile had settled away from the structure it was intended to support. The approximate magnitudes of the forces tending to support the pile and those pushing the pile downward are shown in Fig. 18. According to the log of a boring drilled about 10 ft away from the pile to investigate this failure, the pile tip was driven to the surface of the somewhat decomposed bedrock and probably was crushed against the rock formation. A proper appreciation of the order of magnitude of the potential forces on piling would, in some cases, lead to the selection of a different type of piling or a different type of foundation, whereas, in other cases, it might lead to a decision that the piling should penetrate into firmer strata for a greater distance than would be required by use of a dynamic driving formula.

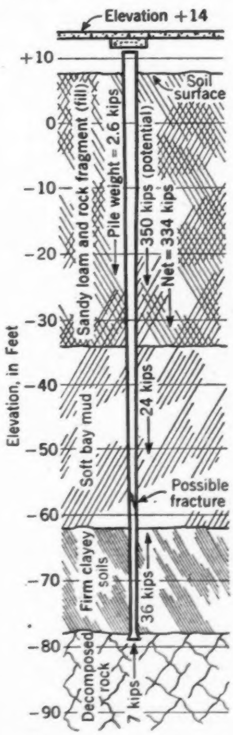

Fig. 18.—Frictional Forces on Pile Shown in Fig. 17; Driving Resistance by *Engineering News* Formula Equaled 27 Tons

Pile foundations have been installed on several scores of projects utilizing information of the nature described. Unfortunately, on many of these projects, it has been impracticable or uneconomical to obtain complete soil data, pile loading test data, driving records, and settlement data. There are, however, a number of projects which furnish fairly complete data on the strengths of the soils and also on full-scale loading tests on typical foundation piling. Table 8 shows comparisons between the capacities of piling as given by the *Engineering News* formula, as indicated by direct full-scale loading tests, and as computed in the fashion described in this paper utilizing data from tests on the soils.

In selecting a suitable type of foundation to support a given structure, consideration must be given to the problems of construction as well as to those of design. If it is not feasible to develop sufficient embedment to provide adequate support for the structure with a given type of piling and driving equipment, the foundation design should be revised, perhaps using a different type of piling, in order to assure adequate and satisfactory support for the proposed

structure. The methods described have been used for several years with satisfactory performance, and better balanced and more economically completed structures have been realized in many cases than would have been achieved through the use of customary dynamic formula control methods.

TABLE 8.—COMPARISONS OF PILE CAPACITY DETERMINATIONS BY DIFFERENT METHODS

Location[a]	Type of pile	Pile length (ft)	Driving hammer	CAPACITY (TONS)		
				Engineering News formula	Load test yield point	Computed temporary
(1)	(2)	(3)	(4)	(5)	(6)	(7)
(*a*) WOOD						
Mare Island		60		7	45	40
Mare Island		70	Vulcan No. 1	14	50	44
Richmond	8-in. tip, 1 in. in 10-ft taper	75		11	80	59
Mare Island		80	McKiernan-Terry	22	25	20
Mare Island		80	10-B-3	25	20	21
(*b*) CAST-IN-PLACE CONCRETE						
San Diego	Fluted metal	60	Vulcan No. 1	43	160	132
San Diego	Step-tapered	60	Raymond special	41	160	148
Compton	Step-tapered	32	Vulcan No. 1	10	33	32
(*c*) PRE-CAST CONCRETE						
Benicia	20 in. square	95	Vulcan OR	21	100	100
Richmond	18 in. × 24 in. tongue and groove	65[b]	Vulcan No. 1	138	90	89
California Point	18 in. square	101	Vulcan No. 0	0	35	42
California Point	18 in. square	101	Vulcan No. 0	24	50	54
(*d*) STEEL H-PILES						
San Diego	14WF87	60	Vulcan No. 1	33	140	128
San Diego	14WF87	60	Vulcan No. 1	37	154	165

[a] All locations in California. [b] Penetration into soil 23 ft.

ACKNOWLEDGMENTS

The procedure presented herein for predetermining pile lengths is protected by United States patent No. 2,296,466 and Dominion of Canada patent No. 413,675 issued in the names of the author and Trent R. Dames, M. ASCE. They, however, offer the results of their experience with the method, together with their patent rights, for the free use of the profession. It is expected, nevertheless, that suitable acknowledgment or recognition will be made by those who use the method.

The writer is most grateful for the generous cooperation of those clients for whom the data presented have been gathered and for their permission to use the data in this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

OPERATION OF THE CONOWINGO HYDROELECTRIC PLANT

By Robert E. Turner,[1] M. ASCE

Synopsis

Large run-of-river hydroelectric (or "hydro") developments on rivers with extremely wide variations in flow can only be justified on a sound economic basis provided such plants can be operated in conjunction with a large steam generating system so as to attain the greatest possible economic use of the hydroelectric power. By reducing the steam generating costs to a minimum, over-all economies can be effected to maximum advantage for the entire system. Allocation of hydroelectric energy to the best advantage requires determinative river flow forecasting, proper use of pondage, maintenance of head, and station efficiency so that maximum capacity and maximum energy can be obtained from the available water to replace the most expensive steam generation. This paper presents in detail the methods by which the various factors are analyzed to determine the most effective operation sequences for a large hydroelectric installation.

Full coordination of hydro and steam capacity in electric power systems has been expanded through interconnected lines on a regional basis to assure fullest utilization of available generating capacity for carrying increased loads, in scheduling maintenance, in meeting emergency outages of equipment, as well as for obtaining economies of generation. Hydro and steam power, rather than being competitive, are actually complementary and many advantages accrue when their use is coordinated in electric power systems.

Conowingo Hydroelectric Development

The Conowingo hydroelectric plant of the Philadelphia (Pa.) Electric System was placed in operation on March 1, 1928. It is located in Maryland on the Susquehanna River about 4 miles above tidewater and approximately

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **April 1, 1948.**

[1] Hydrographer, The Susquehanna Elec. Co., Conowingo, Md.

60 miles from Philadelphia. Each of the seven main generating units installed in the plant consists of a vertical shaft, Francis-type turbine of 54,000-hp capacity, directly connected to a 36,000-kw generator. Energy from the generators is fed to four 80,000-kva, 13.8/220-kv transformer banks, which supply the 220-kv switching station on the roof of the power plant, in turn, supplying two 220-kv circuits.

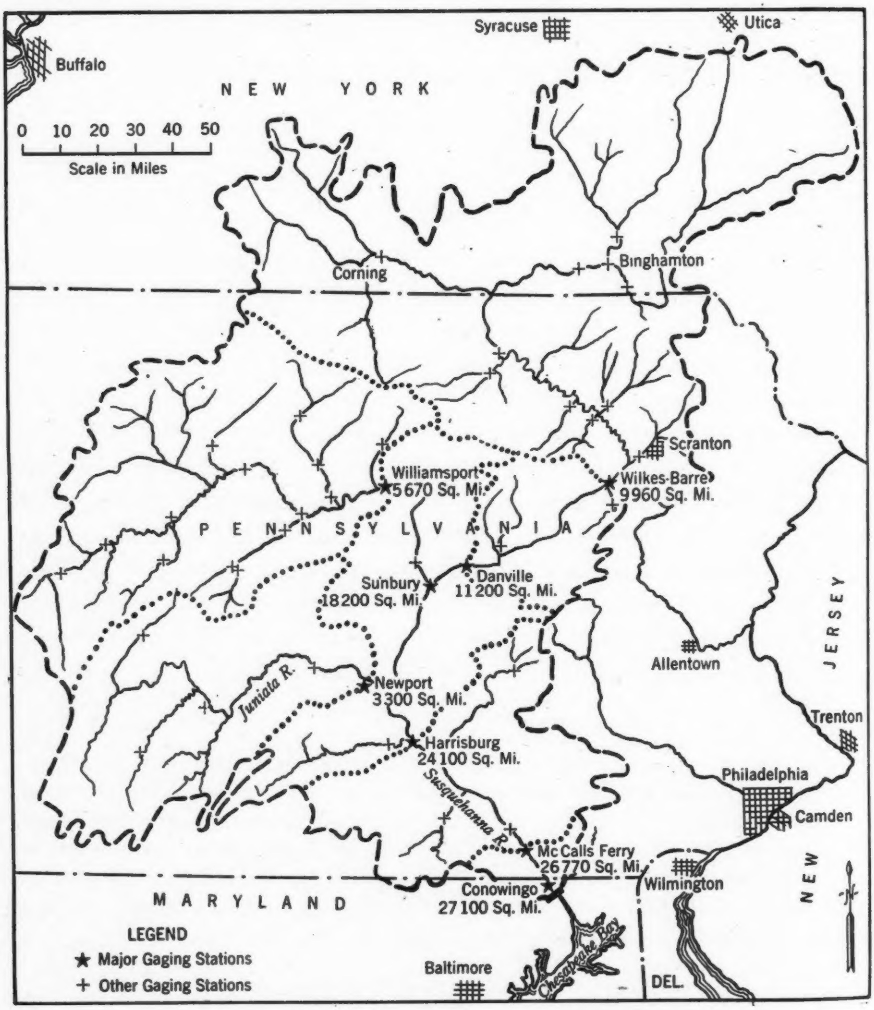

FIG. 1.—GENERAL MAP OF SUSQUEHANNA RIVER BASIN

SUSQUEHANNA RIVER DRAINAGE AREA

The Susquehanna River has a total drainage area of more than 27,000 sq miles, which includes 47% of the total area of the State of Pennsylvania and 13% of the total area of the State of New York, besides a small area in the

State of Maryland (Fig. 1). With the exception of the Saint Lawrence River, in Canada, it is the largest and most important river on the Atlantic coast. The river has rather steep slopes and has very little storage, either natural or artificial.

In its lower reaches, where the river has cut its way through a range of tableland, the river channel has an average slope of almost 6 ft per mile and is walled on both sides by steep, rocky bluffs affording excellent foundation for water-power developments. Four run-of-river hydroelectric plants have been constructed in this stretch. Data for these plants are given in Table 1.

TABLE 1.—Hydroelectric Plants on the Lower Susquehanna River

Plant	Distance from tidewater (miles)	Year of initial operation	Number of units	Head (ft)	Maximum discharge (cu ft per sec)	Effective capacity (kw)	Pondage[a]
Conowingo, Md.	4	1928	7	89	45,000	252,000	4,200
Holtwood, Pa.	19	1910	10	51	32,000	104,000	1,100
Safe Harbor, Pa.	27	1931	7	55	65,000	230,000	3,300
York Haven, Pa.	50	1904	20	20	18,000	20,000	....

[a] Approximate pondage in (cu ft per sec)—days per ft for normal regulation.

River Flow Characteristics

The mean annual precipitation over the drainage area is almost 40 in. and the mean annual runoff averages about 47% of the precipitation. The runoff varies widely from day to day, week to week, and season to season; the recorded minimim, about 2,000 cu ft per sec average for a week, occurred during late summer of 1930, and the maximum recorded flood runoff, about 875,000 cu ft per sec, in March, 1936. The wide variation in runoff throughout the year can be illustrated best by reference to the monthly flow duration curves shown in Fig. 2. It is considered that the Conowingo plant can utilize all flows below 45,000 cu ft per sec. Runoff is at a minimum during August, September, and October, and is at its maximum during March, April, and May.

Interconnected Load

Practically all the output of the Conowingo plant is transmitted over two 220-kv circuits, about 58 miles in length, to the Plymouth Meeting (Pa.) substation located approximately 15 miles northwest of Philadelphia. From there the energy is transmitted into Philadelphia via 66-kv transmission lines and to suburban areas at both 66 kv and 33 kv. Plymouth Meeting substation also serves as the point of tie-in with the Pennsylvania-New Jersey Interconnection at 220 kv. The Pennsylvania-New Jersey Interconnection is made up of the Pennsylvania Power and Light Company, the Public Service Electric and Gas Company of New Jersey, and the Philadelphia Electric Company. Additional interconnection facilities with other companies at different points on these systems makes it possible to supply the large urban and industrial area reaching from New York, N. Y., to the District of Columbia.

The entire output from the Conowingo hydroelectric plant can be used advantageously on the Philadelphia Electric System alone, but actually it is used in combination with the entire interconnected system to reduce steam generating costs to a minimum. The Pennsylvania-New Jersey Interconnec-

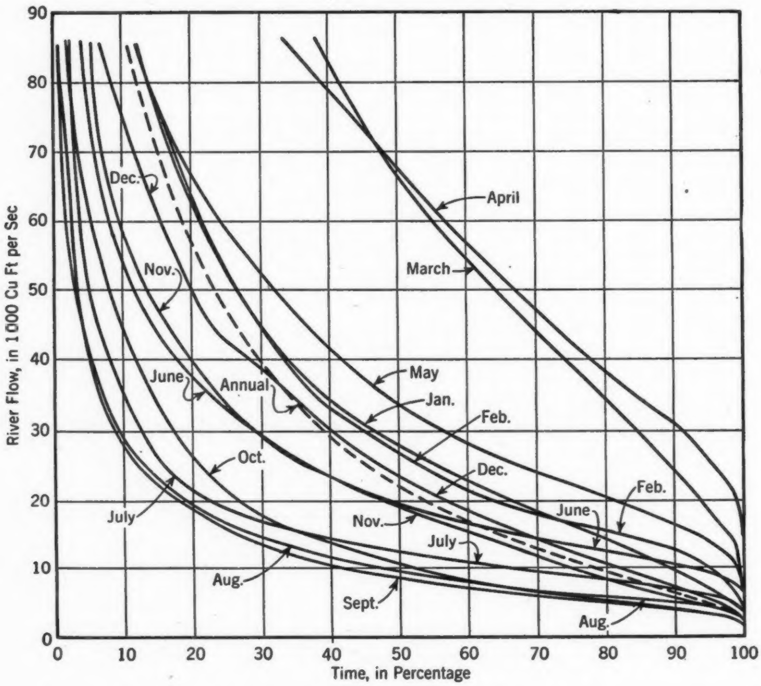

Fig. 2.—Flow Duration Curves of Susquehanna River at Conowingo, Md.

tion has approximately a 3,000,000-kw peak, of which about 300,000 kw can be carried on hydro power. The total area load is about 4,000,000 kw with approximately 650,000 kw hydro power available. The load is served predominantly by steam plants, but efficient design of plants and transmission system, together with close cooperation and no elaborate inflexible contracts, allow the fullest coordination between hydro and steam generating facilities to carry the load at minimum cost.

Allocation of Hydroelectric Energy

The operation of run-of-river hydro plants in a hydro-steam system has been frequently explained. It is sufficient to say that the available hydro capacity should be utilized on the system load so that the available hydro energy is used most effectively, drawing from pondage during the hours and days of heavy demand and refilling during periods of light load. The hydro plant is prepared to deliver up to its maximum capacity and the use of pondage is normally so scheduled as to have the level at top elevation on Monday mornings to meet the load demands of another week and, for all but the lower

flows, top elevation is scheduled each weekday morning, particularly in the early part of the week.

In general, steam generating costs for the system vary directly with the load. Since the hydro production costs are almost constant for all loads, the value of the energy delivered is that of the marginal steam generation it replaces on the interconnected system. The load diagram shown in Fig. 3 illustrates three typical basic conditions for coordination of hydro with steam generation. It can be seen that the operation of equipment in the steam stations is governed, to an appreciable degree, by the estimated output of the hydro station.

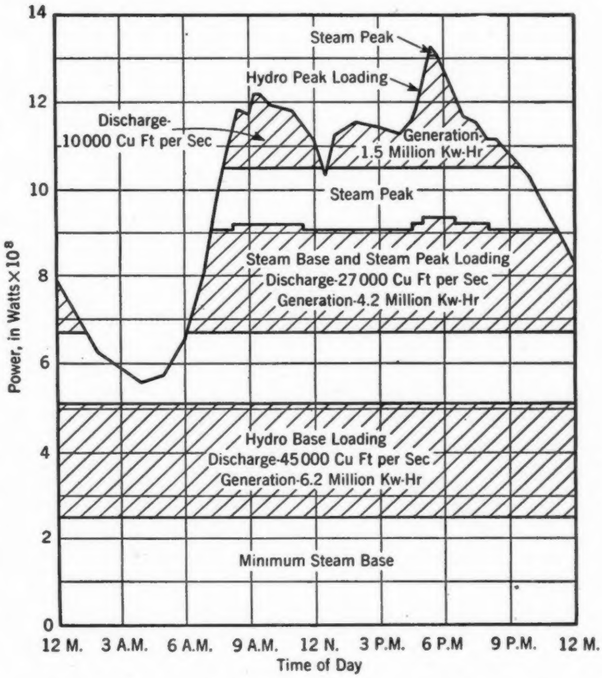

Fig. 3.—Coordinated Use of Hydro Power with Steam Generated Power

With low river flow, the steam base load is carried at an amount where the generation of hydro power over peaks (up to the hydro capacity) will correspond to the available energy from the river flow on a weekly basis—greater on heavy load days, less on light load days, with recuperation of storage over the week end. The available pondage at Conowingo is ample for weekly operation—5,000,000 kw-hr in the top 8 ft, or about double the available weekly energy for the minimum flows of record. Application of the energy to Pennsylvania-New Jersey Interconnection load curves allows fuller use of the available capacity with fewer hours of hydro operation than are indicated in Fig. 3.

When the river flow is greater, the steam base load is decreased so that more hours of hydro operation at greater loads are called for, using the available energy on a daily bais, or for longer periods over week ends. Hydro loads in excess of maximum efficiency on the number of units available may be justified for some periods by the difference in steam generating costs. With high river flows, Conowingo will generate to full capacity on base load 24 hours a day and the steam generating system assumes the burden of providing reserve capacity, of absorbing the variations in load, and of regulating the frequency. Superimposed on the base minimum steam load, and the hydro

base load, the balance of steam generation will vary with the load. With these conditions, the maximum hydro capacity is most important, particularly over the system peak, since higher flows mean higher tailwater elevation and lower plant head, with consequent adverse effect on plant capacity.

The allocation of hydroelectric energy to best advantage requires determinative river flow forecasting, proper use of pondage, maintenance of head, and maintenance of station efficiency so that maximum capacity and the maximum amount of energy can be obtained from the available water to replace the most expensive steam generation. Head and station efficiency are basically designed qualities and can be controlled, however no control can be exercised over river flow, which varies greatly. Knowledge of river flow prospects is the most important consideration in the operation of a hydro plant.

Determinative River Flow Forecasting

It is not difficult to forecast probable river flows when flow is receding, as the recession rate is generally quite definite and, of course, the river flow actually is receding most of the time. To determine the probable increase in river flow, the sequence of meteorological and hydrologic factors must be anticipated progressively.

Active cooperation is carried on with governmental agencies interested in various phases of these problems, particularly with the United States Weather Bureau, which maintains the extensive facilities necessary for recording and gathering information on the air mass and water cycle, and with the United States Geological Survey, which maintains the stream gaging facilities. Some of the same river and rainfall observers are used in the Philadelphia Electric Company's skeleton network for obtaining necessary information. This work is unified through the Federal-State Flood Forecasting Service, a cooperative organization with the Water and Power Resources Board of Pennsylvania, which is unique in the United States and the result, in part, of recommendations made by the writer after the 1936 flood.

Accurate weather forecasts are always valuable for making anticipatory estimates of river flow and for many other uses in the operations of the system. Additionally, the peculiar uses made of specialized river flow forecasts and the particular economic value of such forecast dictate the need for independent river flow forecasting. The general public interest in river flows seems restricted to flood flows and governmental agencies naturally concentrate on these requirements. The interest of the electric system is mainly in the small flows increases and the extremely high floods.

Hydrographs of River Flow.—The method of plotting hydrographs from eleven river observation points on the Susquehanna River and its tributaries above Conowingo is shown in Fig. 4. These hydrographs are not plotted for the actual time of observation; they are transposed by what seems to be the normal time of travel of the crest from these points to Conowingo for the range of flows most valuable (from an economic standpoint) to forecast accurately. In general, the water flows at a rate slightly in excess of 4 miles per hr.

This method of plotting is very useful for forecasting flows when river stages are at crest or approaching their crest, and therefore stabilized, but it

is of little assistance before and during a rain or in the first stages of runoff—an important deficiency in forecasting at a critical time.

Forecasts of Flow Increases.—To meet the deficiency already mentioned, the writer has developed a method for making an initial estimate of flows without stage verification. It is an empirical rather than a theoretical method for the particular conditions most useful for economic operation of the Conowingo hydro plant. While subscribing to the theoretical methods, the writer finds their application best when based on hindsight for actual conditions

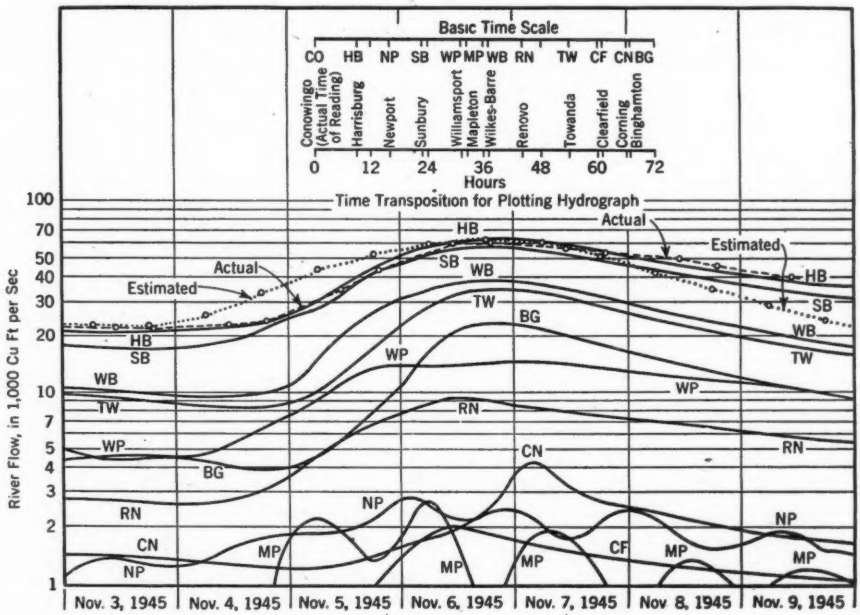

FIG. 4.—HYDROGRAPH OF A FLOW INCREASE ON SUSQUEHANNA RIVER TRIBUTARIES

that have occurred, because in all practical applications insufficient information and lack of time make theoretical methods impracticable. More approximate but quicker practical methods are more useful and valuable for obtaining certain system economies of considerable magnitude.

Variables to Consider in River Flow Forecasts.—Determination of the probable river flows for predicted precipitation or from rain that has just fallen is the most complex, yet the most valuable, forecast of river flow that can be made. The sequence of meteorological and hydrologic factors must be anticipated, progressively, from the development and movement of storms, through rainfall and runoff, to river flow. Some idea of the complexity of such forecasting may be obtained from consideration of the variables involved.

1. Forecasts of Rainfall.—Determinative rainfall forecasting has been improved immensely, but there is still room for further improvement. Full reliance on forecasts of quantity and distribution over the Susquehanna River watershed is not yet justified but such forecasts are of prime importance,

and anticipatory estimates are made for scheduling loads. Close cooperation is maintained with the U. S. Weather Bureau for mutual understanding of the problems.

2. Areal Distribution of Rainfall.—Areal distribution of rainfall cannot be determined accurately, particularly in sections where the terrain is rugged and for summer thunderstorms, unless a large number of reporting stations are used. There is a practical limit to the number of stations used because the task of computation becomes laborious and time-consuming and the additional expense involved is not justified. Most of the reporting stations used are located along the rivers in the valleys. The U. S. Weather Bureau assists in the determination of the average rainfall distribution over the areas.

3. Rainfall-Runoff Ratio.—Because of the many subtractions and delays in transit it is difficult to forecast the amount of surface runoff resulting from a rain. The ratio varies during seasons of the year due to transpiration and evaporation losses, and the absorption and infiltration qualities of the ground. Also important are the effect of antecedent rainfall, the intensity of precipitation, and its duration.

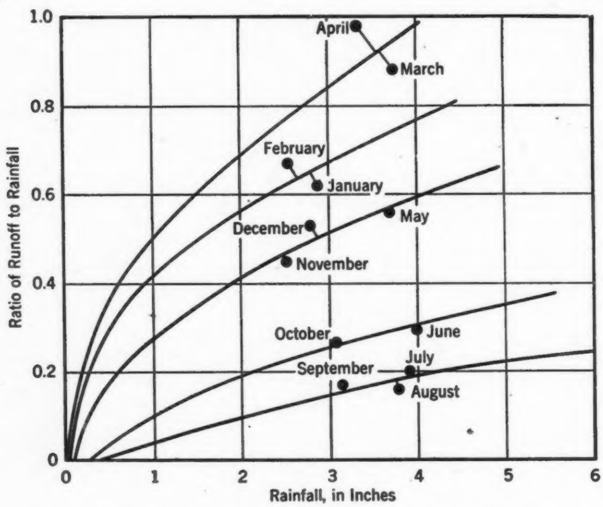

FIG. 5.—RAINFALL-RUNOFF RATIO ON SUSQUEHANNA WATERSHED

A guide for determining the amount of runoff to be expected from a rain during different seasons of the year is shown in Fig. 5. The cumulative rainfall for the storm should be used with these curves to determine the ratio of rainfall which may appear as runoff. No runoff is expected for light rains of different amounts, varying with the season. During wet periods or dry periods, it is necessary to use a higher or lower curve than that shown in Fig. 5 for the month in which the rain occurs, because these ratio curves are based on average conditions. Some judgment must be exercised in choosing the applicable curve. During winter months the runoff can be quite different from that indicated, due to temperature and type of precipitation.

4. Runoff Distribution.—The manner in which the surface runoff flows into the main stream channel depends on the physical characteristics of the drainage area, especially on the size, slopes, and locations of its tributaries. Use has been made of the unit-hydrograph principle that, for the same drainage area, surface runoff from rainfalls occurring within the same time interval, such as a part of a day, will generally produce hydrographs whose ordinates will vary

directly with the total amount of the surface runoff. It is rare, however, that isolated storms occur which produce consistent hydrographs from which an average distribution graph can be determined—the larger the drainage area, the less frequently will they occur. Likewise, it is just as infrequent in actual practice for the storm for which the runoff is forecast to meet the conditions from which the unit hydrograph was derived. Actually, the writer has never seen duplicate conditions result in identical hydrographs from the Susquehanna River basin. Each case must be considered on its individual characteristics.

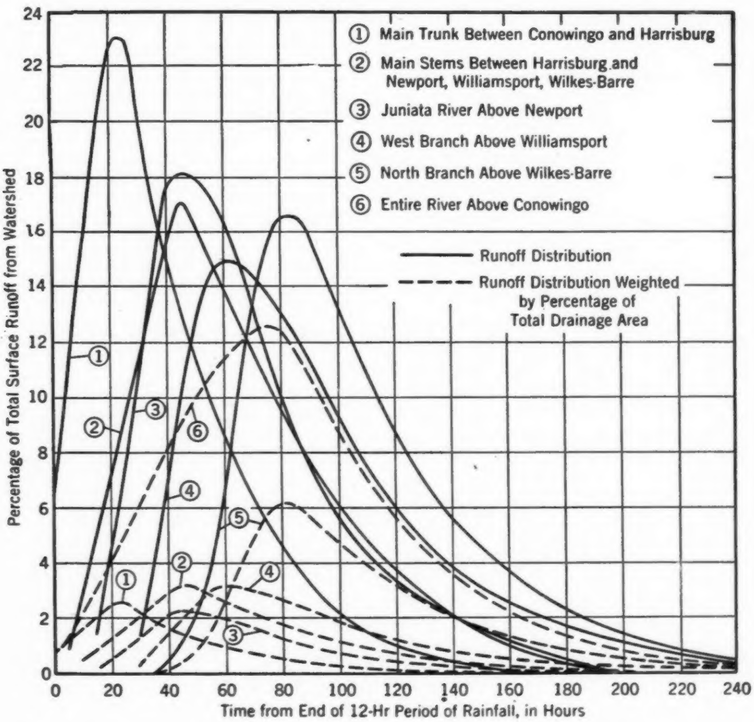

FIG. 6.—RUNOFF DISTRIBUTION CURVES FOR SUBAREAS OF SUSQUEHANNA WATERSHED

The solid curves in Fig. 6 show the normal distribution of runoff, expressed in percentage of total runoff, that may be expected from a 12-hr period of rainfall over the various sections of the watershed as indicated. In plotting they have been transposed in accordance with the basic time scale (Fig. 4) for the water travel time from the various gaging stations to Conowingo. The dashed curves in Fig. 6 show the runoff distribution graphs weighted by the percentage of total drainage area each represents. The sum of the ordinates of these five subarea curves equals the ordinate of curve 6 for the entire watershed above Conowingo. Curve 6, the over-all distribution graph of runoff, is what may be expected when a rainfall of uniform amount occurs over the entire

watershed above Conowingo. The average percentages of total expected runoff contributed by each subarea to the river flow at Conowingo were obtained from the distribution curves (curves 1 to 6) for each 12-hr period after the rainfall by taking the midpoint value for each period. These values are listed in Table 2(*a*) for each reference.

TABLE 2.—DISTRIBUTION OF RUNOFF AND

Sub-area[a]	Total rain-fall (in.)	Runoff-rain-fall ratio	Rainfall in period (in.)	Runoff for period (in.)	Drain-age area $\times K^{(b)}$	Runoff volume for period[c]	NUMBER OF EACH					
							−1[d]	0	1	2	3	4
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
(*a*) PERCENTAGE OF RUNOFF												
1	....	....	....	....	80,400	....	....	2.0	12.0	21.5	20.2	14.0
2	....	....	....	....	137,200	....	....	0	1.4	6.3	11.3	16.2
3	....	....	....	....	90,200	....	....	0	0	3.3	11.0	17.9
4	....	....	....	....	152,800	....	....	0	0	0	1.3	8.1
5	....	....	....	....	267,800	....	....	0	0	0	0	0.7
6	....	....	....	....	728,400	....	....	0	1.5	3.8	6.1	8.6
(*b*) EXPECTED FLOW INCREASE FROM FIRST 12-HR												
1	0.06	0	0.06	0	80,400	0	....	....	....	....	....	....
2	0.34	0.13[e]	0.34	0.044	137,200	6,040	....	0	0.2	0.8	1.4	2.0
3	0.36	0.14[e]	0.36	0.050	90,200	4,520	....	0	0	0.2	1.0	1.6
4	0.59	0.20[e]	0.59	0.118	152,800	18,030	....	0	0	0	0.4	3.0
5	0.53	0.19[e]	0.53	0.101	267,800	27,020	....	0	0	0	0	0.4
Total	....	....	....	....	....	....	....	0	0.2	1.0	2.8	7.0
(*c*) EXPECTED FLOW INCREASE FROM SECOND 12-HR												
1	0.21	0.08[f]	0.15	0.012	80,400	1,000	....	0	0.2	0.4	0.4	0.3
2	0.86	0.25[f]	0.52	0.130	137,200	17,820	....	0	0.4	2.2	4.0	5.8
3	0.76	0.24[f]	0.40	0.096	90,200	8,670	....	0	0	0.6	2.0	3.2
4	1.05	0.34[f]	0.46	0.156	152,800	23,830	....	0	0	0	0.4	3.8
5	0.86	0.31[f]	0.33	0.102	267,800	27,310	....	0	0	0	0	0.4
Total	....	....	....	....	....	....	....	0	0.6	3.2	6.8	13.5
(*d*) EXPECTED FLOW INCREASE FROM THIRD 12-HR												
1	0.24	0.10[g]	0.03	0.003	80,400	240	....	0	0	0.2	0.2	0.1
2	1.05	0.36[g]	0.19	0.068	137,200	9,330	....	0	0.2	1.2	2.2	3.0
3	0.83	0.33[g]	0.07	0.023	90,200	2,080	....	0	0	0.2	0.4	0.8
4	1.14	0.39[g]	0.09	0.035	152,800	5,350	....	0	0	0	0.2	0.8
5	1.24	0.41[g]	0.38	0.156	267,800	41,760	....	0	0	0	0	0.6
Total	....	....	....	....	....	....	....	0	0.2	1.6	3.0	5.3
(*e*) EXPECTED FLOW INCREASE FROM THE THREE 12-HR												
From third period							....	0	0.2	1.6	3.0	5.3
From second period							....	0.6	3.2	6.8	13.5	17.4
From first period							....	1.0	2.8	7.0	10.4	14.2
Total increase in surface runoff							....	1.6	6.2	15.4	26.9	36.9
Expected recession of present flow							23.0	21.1	19.5	18.2	17.0	16.0
Expected future river flow							23.0	22.7	25.7	33.6	43.9	52.9

[a] As described in Fig. 6. [b] $K = 26.89$. [c] In (cu ft per sec)-days. [d] 12-hr period preceding 12-hr rainfall December and January curve (Fig. 5).

It should not be expected that these distribution curves are exact or that actual conditions will be similar to those on which they are based, but it is expected that the accuracy is sufficient for the normal diversity of conditions in the several subareas to give, in the main, a reasonable expectation of the actual flow.

CALCULATION OF EXPECTED FLOW INCREASE AT CONOWINGO

12-HR PERIOD FOLLOWING 12-HR RAINFALL PERIOD												
5 (14)	6 (15)	7 (16)	8 (17)	9 (18)	10 (19)	11 (20)	12 (21)	13 (22)	14 (23)	15 (24)	16 (25)	17 (26)
CONTRIBUTED TO TOTAL FLOW												
10.2	7.2	4.9	3.2	2.0	1.2	0.8	0.5	0.2	0.1	0	....	....
15.2	12.5	10.0	7.7	5.8	4.3	3.2	2.4	1.7	1.1	0.6	0.3	0
17.4	14.6	10.6	7.6	5.4	3.7	2.9	2.2	1.5	0.9	0.6	0.3	0.3
14.2	14.7	13.3	11.3	9.0	6.9	5.3	4.0	3.1	2.4	1.9	1.4	1.4
3.5	10.7	16.2	15.9	12.7	10.0	7.7	5.9	4.5	3.5	2.7	2.0	2.0
10.5	12.0	12.6	11.0	8.5	6.6	5.0	3.8	2.9	2.1	1.6	1.1	1.1
RAINFALL PERIOD (7 A.M. to 7 P.M. 11/2/45)												
....	....	....	....	....	....	....	....	....	....	....	....	....
1.8	1.6	1.2	1.0	0.8	0.6	0.4	0.2	0.2	0.1	0	....	....
1.6	1.4	1.0	0.6	0.4	0.3	0.2	0.2	0.1	0	....	....	....
5.2	5.4	4.8	4.0	3.2	2.4	2.0	1.4	1.2	0.8	0.6	0.6	0.4
1.8	5.8	8.8	8.6	7.4	5.4	4.2	3.2	2.4	1.8	1.4	1.1	0.9
10.4	14.2	15.8	14.2	11.8	8.7	6.8	5.0	3.9	2.7	2.0	1.7	1.3
RAINFALL PERIOD (7 P.M. 11/2/45 to 7 A.M. 11/3/45)												
0.2	0.2	0.1	0.1	0	....	....	....	....	....	....	....	....
5.4	4.4	3.6	2.8	2.0	1.6	1.2	0.8	0.6	0.4	0.3	0.2	0.1
3.0	2.6	1.8	1.4	1.0	0.7	0.5	0.4	0.3	0.2	0.2	0.1	0
6.8	7.0	6.4	5.4	4.2	3.2	2.6	2.0	1.4	1.2	1.0	0.6	0.5
2.0	5.8	8.8	8.6	7.0	5.4	4.2	3.2	2.4	1.8	1.4	1.1	0.9
17.4	20.0	20.7	18.3	14.2	10.9	8.5	6.4	4.7	3.6	2.9	2.0	1.5
RAINFALL PERIOD (7 A.M. to 7 P.M. 11/3/45)												
0	....	....	....	....	....	....	....	....	....	....	....	....
2.8	2.4	1.8	1.4	1.0	0.8	0.6	0.4	0.3	0.2	0.2	0	....
0.8	0.6	0.5	0.4	0.3	0.2	0.2	0.1	0	....	....	....	....
1.6	1.7	1.4	1.2	1.0	0.8	0.6	0.4	0.4	0.3	0.2	0.2	0.2
3.0	9.0	13.6	13.2	10.6	8.4	6.4	5.0	3.8	3.0	2.2	1.7	1.5
8.2	13.7	17.3	16.2	12.9	10.2	7.8	5.9	4.5	3.5	2.6	1.9	1.7
RAINFALL PERIODS (7 A.M. 11/2/45 to 7 P.M. 11/3/45)												
8.2	13.7	17.3	16.2	12.9	10.2	7.8	5.9	4.5	3.5	2.6	1.9	1.7
20.0	20.7	18.3	14.2	10.9	8.5	6.4	4.7	3.6	2.9	2.0	1.5	1.0
15.8	14.2	11.8	8.7	6.8	5.0	3.9	2.7	2.0	1.7	1.3	0.8	0.6
44.0	48.6	47.4	39.1	30.6	23.7	18.1	13.3	10.1	....	....	....	....
15.0	14.2	13.5	12.8	12.2	11.7	11.3	10.9	10.5	....	....	....	....
59.0	62.8	60.9	51.9	42.8	35.4	29.4	24.2	20.6	....	....	....	....

period. [e] From November curve (Fig. 5). [f] From December curve (Fig. 5). [g] From average between

5. Rate of Flow in Streams.—The rate of travel from various points on the river, as indicated by the basic time scale shown in Fig. 4, holds quite well for the flow values that are most critical from the standpoint of economy of use at Conowingo. This time scale must be tempered by judgment when applied to very small or very large increases in flow and the effect of channel storage and intermediate drainage area must be fully considered.

6. Ground-Water Flow.—The foregoing discussion has been concerned only with surface runoff. There is undoubtedly a hydrograph of ground-water flow, but to isolate it and to determine its value has been neither possible nor sufficiently valuable to warrant the effort. Ground-water flow has been likened to a large reservoir discharging through a **V**-notch weir. So far it has not been possible to determine accurate discharge coefficients or an accurate head gage for the simulated weir. Moreover, it is not of major importance in this problem except as to its effect on estimating minimum flows.

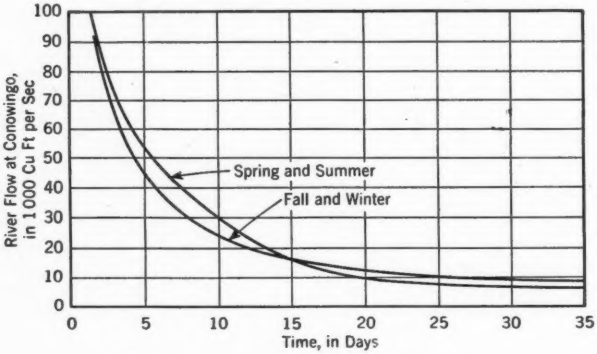

FIG. 7.—NORMAL FLOW RECESSION CURVES AT CONOWINGO

7. Flow Recession.—It is necessary to know the river flow that would have occurred without the new runoff, that is, the base flow upon which the surface runoff is superimposed. Fig. 7 shows the normal flow recession curves for the Susquehanna River when no additional precipitation occurs. These were determined from actual past experience at Conowingo. The values used are read differentially from the curve.

8. Rainfall Along Main Stem of River.—Rain falling directly on the storage ponds and river channels, since its runoff is fast and almost 100%, can have a noticeable effect at low flows. This is particularly true on the main stem of the Susquehanna, when heavy summer thunderstorms travel down the river valley. Occurrence of such rains over week ends can affect operations to a considerable extent.

9. Effect of Temperature.—Temperature can have a large effect on river flows, particularly during the winter months. Precipitation may fall as rain, freezing rain, sleet, and snow. It may be a combination of these or it may be different in various areas of the watershed so that runoff values are indeterminative. Rain may fall on snow and be absorbed, so that runoff may be delayed or even arrested. On the other hand, rain or even warm temperatures alone may bring additional runoff from snow or ice already on the ground. River stages alone must be relied on to verify such indications.

Then, even when the water gets into the river channels, flow can be seriously impeded by severe subfreezing temperatures forming ice, which packs

and jams, thus providing an astonishing amount of channel storage in a comparatively short time. These ice conditions affect the river gages, sometimes giving erroneous river flow readings for a protracted period. Some gages are affected more than others and the result is that true gaging is not possible in some cases—in fact, the only true measure is through the powerhouse units.

Calculation of Expected Flow Increases.—The method of calculating the expected increase in flow, either from forecast rainfall or from rain that has actually fallen, or both, is shown on the work sheet reproduced in Tables 2(*b*), 2(*c*), 2(*d*), and 2(*e*). A freshet occurring from November 3 to November 10, 1945, is used because it represents a separate clear-cut computation. The procedure is

1. Obtain the average rainfall for each subarea of the watershed for each 12-hr period, usually ending at 7 a.m. and at 7 p.m.
2. Obtain the ratio of runoff to rainfall from the curves shown in Fig. 5, considering the total rainfall for the periods immediately preceding, as well as the current period.
3. Multiply this ratio by the rainfall for the present period to get the expected runoff in inches. Multiplying this value by the drainage area in square miles and a constant of 26.89 will give the total runoff volume in (cubic feet per second)-days from this period rainfall.
4. Multiplication of this total runoff volume, for the area concerned, by the respective percentages from Table 2 gives the expected increase in total surface runoff contributed by this area for each subsequent 12-hr period. Multiplication of this runoff value by 2 determines the rate in cubic feet per second.
5. Addition of the runoffs thus obtained for each of the five drainage subareas gives the total additional surface runoff rate in cubic feet per second for each subsequent 12-hr period because of the rain that has fallen in the present period.
6. Addition, in proper succession, of these separate 12-hr period runoff rates gives the total increase in surface runoff in average (cubic feet per second)-days resulting from the storm.
7. Addition to this increase in surface runoff of the base recession flows corresponding to the flow occurring at the time of the precipitation (in some cases the increase in surface runoff only serves to counteract the natural recession if no rain has occurred) will result in the expected average rate of flow, for this particular freshet, of each subsequent 12-hr period after the rainfall measurement.

An actual example, plotted as short dashed lines on the hydrograph (Fig. 4), is the estimate made on Saturday evening, November 3, 1945, immediately after all rainfall had occurred. The probable effect of channel storage will be noted as it is more marked than usual in this case.

Appraisal of Method.—During 1945, which was a record output year for Conowingo, there were a number of occurrences when probable flow increases necessitated just such calculation as given by this approximate method. It cannot be expected that such an approximate method of calculation will

always result in accurate flow estimates—first, the method was developed only for the range of river flows most important, from the system economy standpoint, to forecast quickly without stage verification; and, second, no method of calculation can be any better than the ability to determine the variables previously discussed.

Determinative forecasts of river flows is a complex problem that defies definite computation within practical limits of time and expense. Nevertheless, some basis for computation is most valuable, and good judgment obtained from experience, exercised and tempered with proper consideration of the probable effect on system economy, is an important ingredient.

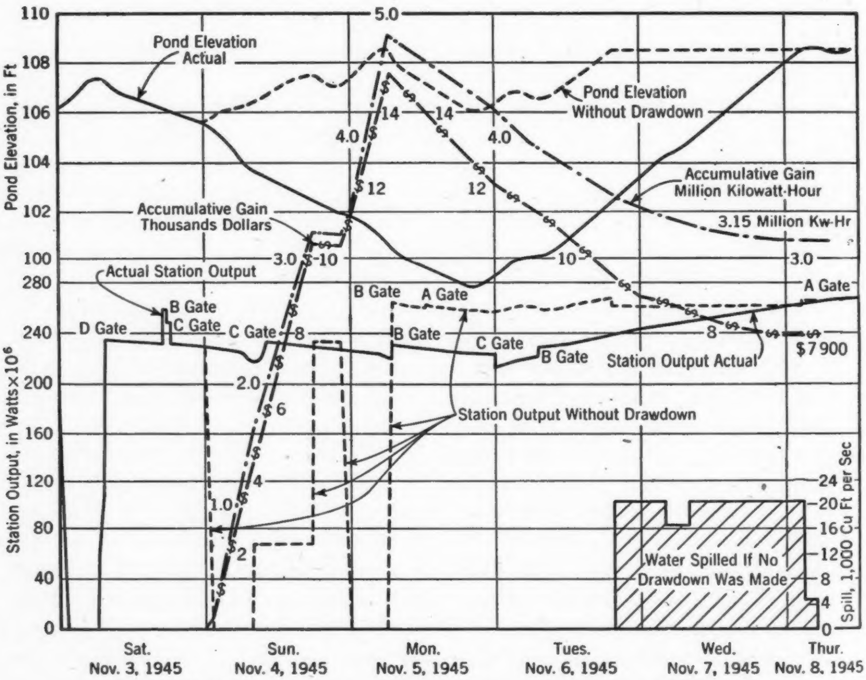

Fig. 8.—Comparison of Actual and Alternative Methods of Operation

Effects of System Economics.—That system economics play a large part in determining what course of action to take can be best illustrated by an actual case of operation. The basic operating information for the freshet occurring from November 3 to November 10, 1945, is shown in Fig. 8. Fundamentally, two courses of action were open on Saturday, November 3, 1945: Either to draw down the Conowingo pond, using the water therein to generate hydroelectric energy to displace low cost steam energy over the week end and sacrificing head and therefore energy and capacity on Monday and Tuesday, when steam replacement costs were higher, or, to carry the hydroelectric energy available from the river flow with recovery of storage by Monday morning, then operating at full output thereafter. After weighing the probable disadvantages of reduced capacity on Monday and Tuesday, the first alternative

was chosen. This was the proper choice, and resulted in a net saving of approximately \$7,900.

As a case in point, this precipitation could have occurred two days earlier in the week. The value of the drawdown would then have been greater on Thursday and Friday and the penalties from low head would have been less on Saturday and Sunday, resulting in a greater economic gain. Conversely, heavier loads on Monday and Tuesday, or forced outages of steam equipment, might have raised steam replacement costs to a point where such a drawdown would show little if any gain; or, more important, if the flow increase had not been great enough to fill the storage, a net loss, rather than a gain, would have resulted from the low head.

In any event, good estimates of river flow at the earliest possible moment, even based on predicted precipitation, may be vital if certain system economies of considerable magnitude are to be obtained or if uneconomical operation is to be avoided.

Maintenance of Head

After the determination of available river flow, the next most important considerations are the maintenance of maximum head and the maintenance of maximum station efficiency, so that the greatest possible capacity and energy may be available for replacement of the highest cost steam generation. These two considerations are interrelated in many ways.

Seiches in the Conowingo Pond.—By agreement with the owners of the Holtwood (Pa.) plant prior to the construction of the Conowingo development,

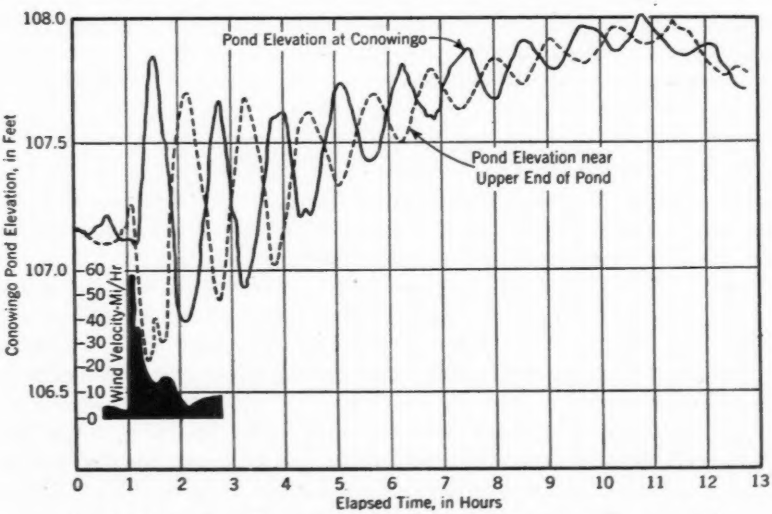

Fig. 9.—Example of Seiche on Conowingo Pond Caused by High Wind

a maximum elevation of 108.5 ft above mean sea level was adopted for the Conowingo Dam pond with the anticipation that it would be the normal elevation. However, from the standpoint of system economy, it is not desirable and, from a practical standpoint, it is not possible to maintain a fixed

elevation. If 108.5 ft is the maximum elevation, then the average must be less, even when operating on base load and spilling water.

The Conowingo pond level has a regular oscillation whenever the equilibrium is upset, such as sudden increase or decrease of load on Conowingo, sudden change in discharge from the Holtwood plant, failure of the Holtwood flashboards under high flow, raising or lowering of spillway gates at Conowingo, or wind. A clear-cut occurrence caused by a sudden squall shortly after 8 p.m. on a summer day is illustrated in Fig. 9. A wind approaching 60 miles per hr blew almost directly downriver along the longitudinal axis of the pond, pushing ahead of it a wave of increasing size, which was followed by a trough of similar magnitude. The wave registered first on a gage near the upper end of the pond, before progressing downstream to the dam, from which it was reflected. The conditions presented an opportunity to determine the natural oscillation period of the Conowingo pond, which proved to be about 1 hour and ten min. In this instance the strong wind gust lasted about one quarter of the oscillatory period, which undoubtedly assisted in setting up a uniform oscillation of considerable amplitude. Knowledge of the natural period of the pond has been helpful to the operators in determining the probable effect of spillway gate operations so as to maintain high average elevation under the limiting maximum elevation.

Effect of Spill Water on Head.—During spill periods, station capacity is curtailed because of increased tailwater elevation. It has been valuable to know where along the dam this excess water may be spilled to cause the least increase in tailwater elevation at the plant for all surplus flows. Also, to avoid opening additional spillway gates at the time of the system peak or even to be able to close some at that time may be very desirable if the "peak prepared for" steam costs are high. For this purpose, some drawdown of the pond just prior to the time of the system peak may be justified.

Maintenance of Station Efficiency

Determination of the available river flow and maintenance of head are joined by the third important consideration, which is the efficiency of the station in converting the water power into electrical energy.

Unit Loading.—In maintaining maximum station efficiency, the unit loading is important. Operating characteristics of the various units have been established on the basis of tests of the individual machine. Periodically, check tests are made to detect any variation in performance from established standards.

If the economy of the entire system, steam and hydro, is not otherwise improved, the loading of Conowingo is such as to secure the highest hydro equivalent (kilowatt/cubic feet per second) consistent with the use of all the available water. To accomplish this, the minimum number of units are operated to meet the load demand and these units are normally not loaded in excess of maximum efficiency, the balance of their capacity being available for reserve. For loading in excess of maximum efficiency, whether to take care of higher river flows or to meet greater load demands, four different turbine gate settings have been established on an incremental basis to cover prac-

tically all conditions of load on seven units. The details of these settings are given in Table 3.

TABLE 3.—LOADING CHARACTERISTICS ON CONOWINGO UNITS

Turbine gate setting (1)	Station output (kw) (2)	Plant discharge[a] (cu ft per sec) (3)	Hydro equivalent[a] (kw per cu ft per sec) (4)	Incremental hydro equivalent from next lower gate setting (5)
A.......	267,000	45,400	5.88	1.13
B.......	264,500	43,000	6.15	3.30
C.......	250,000	38,600	6.47	4.81
D.......	236,500	35,800	6.60	....

[a] For 88-ft plant head.

A (maximum output) gate, giving the maximum possible output (Conowingo turbines are all over-gated), is used only when spilling water or when such possibility is certain to occur in 24 hours, and to meet unforeseen load demands.

B (maximum drawdown or peak output) gate is the normal limit when there is no imminent wastage of water or unforseen load demands.

C (economical limit) gate is considered the economical limit for purposes of obtaining energy other than for load peaks or for maximum drawdown in advance of a freshet. Unit efficiency drops rapidly above this point.

D (maximum efficiency) gate is the most economical point on all units.

The higher, more inefficient gate openings are used for the periods during the day when, by sacrificing Conowingo economy, greater monetary gains can be made on the system, or to take care of river flows greater than can be utilized by 24-hr operation at maximum efficiency. It can be seen from Table 3 that if the steam peak costs exceed the steam base costs by about 40% there is economic justification for operating at C gate instead of at D gate. If the steam peak costs are more than twice the steam base costs, B-gate operation is justified. The small amount of additional generation between B gate and A gate at a cost ratio of 6 to 1 limits its use to spill periods when the water may otherwise be wasted.

Use of Pondage.—Care must be exercised in the use of pondage by drawdown, as it is possible to cause head losses of considerable magnitude, reducing not only the available capacity (approximately 4,000 kw per ft), but also the energy generated from the water. This reduction in energy amounts to about $1\frac{3}{4}$ kw-hr per (cu ft per sec)-day per ft. In effecting economies on the connected steam system, Conowingo may be so loaded as to reduce the capacity requirements of the connected steam system or to absorb the daily system load variations, which may be the result of differences in the normal demand from day to day, either unforeseen or forecasted. If there is an overdraft on the Conowingo pond for any cause, replacement of the storage is effected as soon as possible by reducing the demand on Conowingo. This reimbursement is made, if possible, when the marginal cost of the connected steam system is less than it was when the drawdown was made.

1. Scheduled Drawdowns.—In general, scheduled drawdowns for the purpose of system economy are made late in the week to minimize head losses and capacity reductions when full capacity may be needed; the pond then is refilled over the week end when increment steam costs are lower. The available hydro generation is estimated for the 3 or 4 days remaining in the week and the daily output is then prorated to obtain such savings in the operation of the steam stations as difference in incremental steam generating costs, and reduction in boiler and machine hours. Generally, the greatest saving by drawdown is obtained when the steam base can be held the same for the days for which the energy estimates are made. The coordinated use of hydro storage allows maintenance outages of steam equipment to be scheduled to best advantage.

2. Drawdowns in Anticipation of Increased Flow.—When higher flows are expected, drawdowns may replace steam generation, sacrificing head at Conowingo to obtain the greatest monetary gains on the system. With well-defined possibilities of river flows in excess of the usable flow at Conowingo, full drawdown of the storage is begun and energy is generated by water that might otherwise be wasted over the spillway. It might be well to point out in this connection that the Susquehanna watershed is peculiarly situated, in that the normal paths of storms causing heavy precipitations are such as to result in heavy rains on the lower part of the watershed before the upper sections are affected. It is, therefore necessary to start a complete drawdown, if such is desired, even in advance of the rainfall, otherwise local inflow might make drawdown impossible. Then, if the expected flow increases do not materialize, due to meteorological changes, a quick correction is necessary to minimize head losses. A good gamble, with odds, is worth taking.

It is sometimes difficult to visualize the fact that drawdowns made in advance of excess water are not always profitable, because it appears that energy obtained from use of pondage that is later refilled by otherwise wasted water is an obvious gain. This, however, is not always true. Drawdowns are necessarily made by replacing "off peak" (and hence lower cost) steam generation, yet the head losses are even more effective on high cost than on low cost steam generation. Even without this consideration of difference in increment costs, the actual gain in kilowatt-hours alone diminishes quite rapidly for drawdowns when river flows are moderate or greater. Also, the higher the inflow, the less the drawdowns can be without taking undue risks.

An attempt to illustrate the interrelation of drawdown and possible gain at various flows is shown in Fig. 10. The four top curves show the energy available in the pondage when used through the Conowingo units without considering the losses from any other inflow which would be used at the lowered head. The three bottom curves show the losses in energy obtained from three different values of inflow, because of the loss of head, as drawdown of storage is made. These inflows are expressed in (cubic feet per second)-days to take into account the time element involved. The dashed curves show the net gains possible from drawdown, using most efficient turbine gate openings and with inflows of three different values. If the rate of inflow was 24,000 cu ft per sec, then all drawdown would be made off peak and operation at C gate would be justified most of the day. A curve of net energy at C gate is, therefore, also

shown at this flow. Likewise, if the rate of inflow was 36,000 cu ft per sec, drawdown would have to be made at a higher gate so a curve of net energy at B gate is also shown for this flow.

These curves show that possible energy gains are greatest at the low flows and so attempts are made to reach the drawdown limit unless capacity is too valuable to risk. For higher inflows, drawdowns below El. 100 are rarely

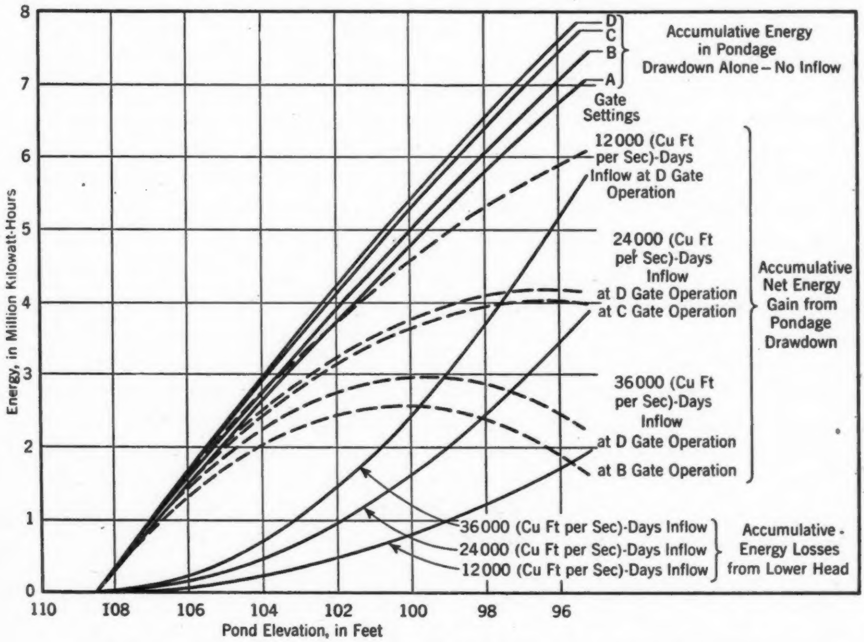

FIG. 10.—INTERRELATION OF DRAWDOWN AND ENERGY GAIN FROM CONOWINGO

justified and in some cases monetary losses may result due to difference in incremental steam generating costs and capacity value. Of course, inability to forecast increases in flow far enough in advance to make possible these large drafts on pondage, particularly with the higher flows, limits the chances of making unprofitable drawdowns if care is taken to consider the replacement costs on steam.

System Reserve Capacity Requirements.—Hydroelectric generating plants have a peculiar value for system reserve. Energy is always available in pondage for immediate use and at times system requirements may dictate having more units running, for reserve purposes, than the load demands. These extra machine hours are kept at a minimum because the no-load water use is appreciable during low flows. Further economy can be effected by placing these generators in condenser operation by closing the turbine gates and, if tailwater level approaches the turbine level, by depressing the water level by the use of compressed air injected into the turbine head cover. Naturally, all unit capacity in excess of the gate opening at which each is operating is available immediately for system reserve and if additional units are required and available they can be synchronized very quickly.

Upriver Hydro Plant Operation

Pondage at the upriver hydroelectric plants (see Table 1) is an important factor and the interrelation of operation with that of the upriver plants is given careful consideration in order to eliminate excess pond elevations or head losses. If conditions are critical, river flow estimates are compared. Close cooperation and an understanding of all problems with which there is mutual concern have resulted in the most efficient utilization of river flow.

Coordinated operation of hydro and steam capacity in electric power systems on a regional basis, through interconnected tie lines, has been expanded to assure fullest utilization of available generating capacity in carrying increased loads, in scheduling maintenance, and in meeting emergency outages of equipment, as well as for obtaining economies of generation. An excellent presentation of the results of cooperative studies for coordinated operation of the hydro plants on the Susquehanna River with the large regional steam generating system has been published elsewhere.[2]

Estimates of Conowingo Generation

Estimates of energy available from Conowingo are needed for more than just a day in advance to obtain maximum economies in the system. Whereas day to day estimates are most important, weekly estimates are necessary to take advantage of incremental costs in scheduling less inefficient steam stations, particularly during low flow. Biweekly estimates are helpful in order that shipments of coal may keep pace with steam generating station demands. Monthly and even yearly estimates are helpful for budget purposes, as large deviations from the average Conowingo outputs affect the cash position of the company, as well as economies in scheduling maintenance. Naturally, the longer the period the less reliable these estimates are.

Cooperative Supervision

Control of the supply of power to meet the demand on all sections of the system is delegated to the load dispatcher and, therefore, allocation of load to attain the greatest economy must be under the same centralized control because the system load and the available sources of energy are subject to continual change. Because the Conowingo output affects system economy so definitely, it is as important sometimes to indicate possibilities as it is to actually forecast their occurrence. For this reason a quick method of determining river flow, as explained herein, is needed. Exchange of information regarding possible changes of system load and equipment outage is helpful to the forecaster as indicating what probabilities of economy may exist. It is imperative to maintain close cooperation and mutual understanding between the responsible parties so that, by constant vigilance, over-all economies can be effected to maximum advantage and uneconomical operation avoided. Hydro and steam power are not rivals, as is popularly believed, but are actually complementary and can be advantageously used together in a coordinated system for over-all economy.

Acknowledgments

The author desires to express his thanks to the executives of the Philadelphia Electric Company for their courtesy in permitting use of the material contained herein, and to his associates in the company for their constructive criticism and assistance in assembling the data.

[2] "The Co-Ordinated Operation of Hydro and Steam Capacity in Electric Power Systems," by G. W. Spaulding, *Transactions*, A.S.M.E., Vol. 66, 1944, p. 545.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

DETERMINATION OF POSITION AND AZIMUTH BY SIMPLE AND ACCURATE METHODS

By T. F. Hickerson,[1] M. ASCE

Synopsis

Determination of latitude, longitude, and azimuth by stellar observations is outlined in this paper by simplified methods employing only the secant and cosecant functions. Also presented is a discussion of the effect of variations, or possible errors, in certain elements of the astronomical triangle, from which may be drawn certain conclusions as to the most favorable conditions for accuracy in the determination of latitude, longitude, and azimuth.

Introduction

The surveyor or engineer aims to establish the azimuth of a line, or to locate the true meridian, to the nearest minute or fraction thereof. The marine navigator, on the other hand, may plot his lines of position satisfactorily when the azimuths are accurate only to the nearest degree, and by the analytical method, as given herein, may determine his position (latitude and longitude) without knowing azimuth. This paper, therefore, is presented primarily for the consideration of the engineer and only incidentally for that of the navigator.

In all cases of precise azimuth determination the latitude must be known closely unless the observed body is on or near the 6-hr circle, as is shown hereafter. The longitude must also be known, unless the body is at or near elongation. Hence, to be sure of accurate azimuths it is desirable that the engineer be able to determine latitude and longitude independently and that he not be forced to rely on obtaining these data from maps—which may not be available.

True azimuths are frequently found from solar observations, but these readings are subject to errors due to two main causes. The first of these is that the sun may be in unfavorable positions; the second, that the average of readings on the upper and lower limbs and on the right and left limbs may fail appreciably to give what is desired—namely, a representative horizontal and

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **April 1, 1948**.

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vertical circle reading at the known instant the line of sight is directed to the sun's center.

The number of so-called "navigational" stars that are also circumpolar at a particular latitude is quite limited. Among the bright and easily identified circumpolar stars always visible at many places in the northern hemisphere are Capella, Vega, Aldebaran, and Regulus. These stars are much brighter than Polaris, in fact they may be visible when Polaris is obscured by haze or by a cloud. For observation in the southern hemisphere three prominent circumpolar stars with large declinations are Acrux, Canopus, and Sirius, the last being the brightest star in the sky.

Too often engineers wait until the early hours of the morning to observe Polaris at elongation, whereas other stars might serve the same purpose at a convenient time.

Notation

The following letter symbols and definitions applying to the astronomical or celestial triangle (triangle PZS in Figs. 1, 2, and 3) are adopted for use in the paper and for the guidance of discussers.

A = a substitution factor = log cosecant $\times 10^5$;
B = a substitution factor = log secant $\times 10^5$;
F = point of intersection of observer's meridian with perpendicular from point S to the meridian;

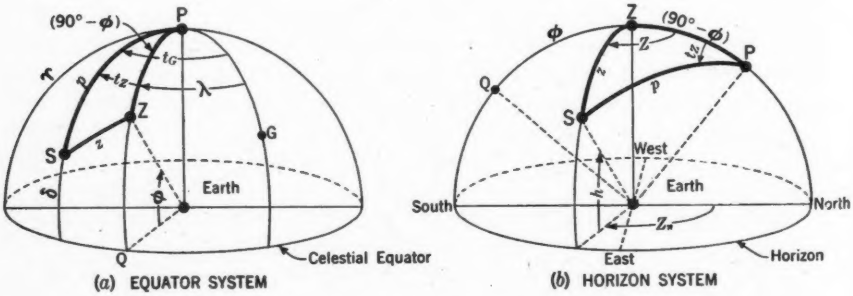


Fig. 1.—Relation of Points and Angles in Celestial Coordinate Systems

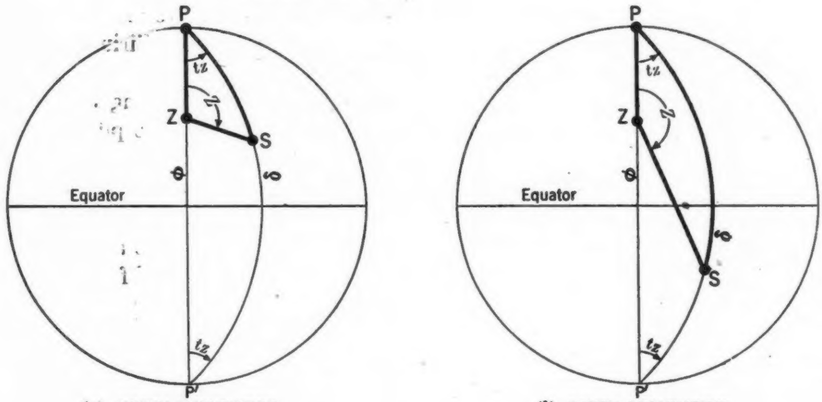
G = Greenwich (reference point);
h = altitude; angle of elevation to the sun or star:
h_o = observed (corrected);
h_s = as given by surveyor's transit or sextant (uncorrected);
P = projection of the north (or south) pole on the celestial sphere;
p = polar distance = $90° - \delta$;
Q = intersection of observer's meridian with celestial equator;
S = projection of the sun or star on the celestial sphere;
s = substitution factor = one half the sum of the sides of a triangle;

t = the hour angle:

t_G = at Greenwich, measured westward from the Greenwich meridian to the sun or star;

t_S = sidereal, measured westward from vernal equinox to sun or star;

t_Z = local, measured westward from the observer's meridian to the sun or star;



(*a*) NORTH DECLINATION (*b*) SOUTH DECLINATION

Fig. 2.—Relation of Points in PZS Triangle for Sun or Star in Eastern Sky

Z = the zenith at the observer's position;

Z = azimuth angle, reckoned from 0° to 180° east or west of the observer's meridian (sun or star in eastern or western sky); Z_n = azimuth, reckoned from 0° to 360° clockwise from north;

z = zenith distance = $90° - h$;

α = right ascension of sun or star = $360° - t_S$;

γ = arc SF (Fig. 3); perpendicular distance from S to the observer's meridian;

δ = declination; position of the sun or star north or south of the celestial equator;

λ = longitude; the observer's position east or west of the Greenwich meridian;

ϕ = latitude; the observer's position north or south of the equator; ϕ_F = latitude of point F (Fig. 3) north or south of the equator;

Υ = vernal equinox (reference point for stars).

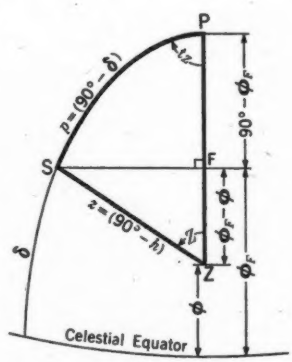


Fig. 3.

Determination of Latitude

Well-known methods of determining latitude are: (1) By meridian altitude of the sun (noon sight); (2) by altitude of Polaris; and (3) by meridian altitude of a star.

Another method applying more readily to either hemisphere is presented here. In Fig. 3 is shown the PZS triangle, in which arc SF ($= \gamma$) is drawn from S perpendicular to arc PZ at F, thus forming right triangles SFP and SFZ. Two cases arise, depending on whether the perpendicular SF falls inside or outside the triangle. If ϕ_F is the latitude of point F, it follows that $(\phi_F - \phi)$ is positive if the perpendicular is inside the triangle, and negative, if outside. Only the first of these two conditions is shown herewith.

Application of the Napier rules to Fig. 3 gives:

$$\sin \gamma = \sin t_Z \cos \delta \qquad (1)$$

$$\sin \phi_F = \frac{\sin \delta}{\cos \gamma} \qquad (2)$$

and

$$\cos (\phi_F - \phi) = \frac{\sin h}{\cos \gamma} \qquad (3)$$

Since the cosecant and secant are reciprocals of the sine and cosine, respectively, these formulas may be rewritten:

$$\csc \gamma = \csc t_Z \sec \delta \qquad (4)$$

$$\csc \phi_F = \frac{\csc \delta}{\sec \gamma} \qquad (5)$$

and

$$\sec (\phi_F - \phi) = \frac{\csc h_o}{\sec \gamma} \qquad (6)$$

It follows that ϕ can be computed if t_Z, δ, and h_o are known. The quantity $(\phi_F - \phi)$ may be positive or negative. It is positive if the body is in the northern sky and negative if the body is in the southern sky. The sign of ϕ_F is the same as that of δ.

$t_G =$	49°26.5′		A = log csc × 10^5	
$\lambda =$	48°00.0′		B = log sec × 10^5	
$t_Z =$	1°26.5′ (approximately)			
		ADD	SUBTRACT	SUBTRACT
$h_o =$	59°27.2′ →			A 6489
$t_Z =$	1°26.5′ →	A 159930		
$\delta =$	5°21.8′ →	B 191 →	A 102932	
$\gamma =$		A 160121 →	B 14 →	B 14
$\phi_F =$	5°21.9′		← A 102918	
	Subtract			
$(\phi_F - \phi) =$	− 30°31.0′[a]			← B 6475
$\phi =$	35°52.9′N = Latitude			

[a] $(\phi_F - \phi)$ is negative because star is in the southern sky.

FIG. 4.—WORK FORM FOR EXAMPLE 1

Later it will be shown that favorable positions of the body for accurate latitude determination occur when it is on or near the observer's meridian. In the example which follows, the star Procyon is in the southern sky shortly after it has crossed the observer's meridian. (The time of meridian transit of all navigational stars for any date is found in the nautical almanac.)

Example 1.—On April 10, 1946, at zone time 18 hr 40 min 12 sec, the corrected altitude to the star Procyon is 59°27.2′. The dead reckoning position of the observer is latitude 36°20′N, longitude 48°00′W. The Greenwich hour angle t_G of Procyon is 49°26.5′ and its declination δ is 5°21.8′. It is required to

calculate the correct latitude by means of Eqs. 4, 5, and 6. The work form shown in Fig. 4 is believed to be self explanatory.

Determination of Longitude

Time, or longitude, may be determined when the declination, altitude, and latitude are known, the latitude having been found previously from pointings to bodies on or near the meridian. Also, if the observer is on a ship or plane, this latitude would be combined with a determination made by dead reckoning. It will be shown later that observations on bodies on or near the prime vertical—that is, east or west of the observer, lead to accurate values of the local hour angle t_Z or the longitude λ, even when the latitude is approximate.

The local hour angle t_Z from solar or star observations may be calculated accurately, and without rules as to quadrant, from the formula:

$$\sec^2 \tfrac{1}{2}\, t_Z = \frac{\sec s \sec (s - z)}{\sec \delta \sec \phi} \qquad \text{(7)}$$

in which $s = \frac{1}{2}(z + \delta + \phi)$. Eq. 7, which is unusual in that it requires the use of only one trigonometric function, the secant, whose logarithm is always positive and greater than unity, was derived from

$$\cos^2 \tfrac{1}{2}\, A = \frac{\sin s \sin (s - a)}{\sin b \sin c} \qquad \text{(8)}$$

—which applies to a spherical triangle whose angles are A, B, and C and whose opposite sides are a, b, and c. By letting $A = t_Z$, $a = 90° - h$, $b = 90° - \delta$, and $c = 90° - \phi$; and by substituting secants for reciprocals of the cosine, Eq. 8 is transformed into Eq. 7, which represents a solution of the astronomical triangle (Fig. 2).

Example 2.—At sea, on August 26, 1946, at 7 hr 30 min a.m., the corrected altitude to the sun's center is 24°12.0′. The dead reckoning position of the ship is latitude 28°34′N, longitude 44°20′W. The Greenwich hour angle of the sun is 337°01.5′ and its declination is 10°33.6′. It is required to calculate the longitude by use of Eq. 7. The work form shown in Fig. 5 is self explanatory except that the preliminary additions and subtractions can be checked before entering the table (a table of log secants), since $(s - z) + (s - \delta)$ always gives ϕ.

$B = \log \sec \times 10^5$

h_o =	24°12.0′	B
z =	65°48.0′	
δ =	10°33.6′ →	742
ϕ =	28°34.0′ →	5638
$2s$ =	104°55.6′	6380 →
s =	52°27.8′ →	21519
$(s - z)$ =	− 13°20.2′ →	1187
$(s - \delta)$ =	41°54.2′	22706
ϕ =	28°34.0′ (check)	6380 ←
		2)16326
$\frac{1}{2} t_Z$ =	− 34°02.4′	← 8163
t_Z =	− 68°04.8′	
t_Z =	291°55.2′	
t_G =	337°01.5′	
λ =	45°06.3′ (= west longitude)	

FIG. 5.—WORK FORM FOR EXAMPLE 2

Determination of Azimuth

Precise determination of the meridian or the true azimuth of a line is, or should be, an integral part of all important land survey and development operations. In making this determination three conditions are encountered. The first is that in which δ, ϕ, and h_o are given and it is required to find Z.

Replacing t_Z with Z in Eq. 7 and substituting the proper values from Fig. 1(*b*) results in—

$$\sec^2 \tfrac{1}{2} Z = \frac{\sec s \sec (s - p)}{\sec h_o \sec \phi} \qquad (9)$$

—in which Z is the azimuth angle reckoned east or west of the observer's meridian depending on whether the body is in the eastern or western sky, and $s = \frac{1}{2}(p + h_o + \phi)$.

Like Eq. 7, Eq. 9 involves only one trigonometric function, the secant. Since Z is never near 0°, but always about 90°, $\frac{1}{2} Z$ will be approximately 45°. Hence, Eq. 9 is satisfactory for all cases that arise.

Example 3.—On July 31, 1946, at 10 hr 00 min p.m. Eastern Standard Time, the corrected altitude to the star Arcturus, in the western sky, is 34°35.0′. The observer's position is latitude 35°10′N, longitude 79°40′W. The star's declination is 19°27.8′N. The work form for the calculation of the azimuth by Eq. 9 is shown in Fig. 6. Here again the preliminary additions and subtractions can be checked before entering the table (a table of log secants). This is true because $(s - p) + (s - h_o) = \phi$.

$B = \log \sec \times 10^5$

		B
$\delta =$	19°27.8′	
$p =$	70°32.2′	
$h_o =$	34°35.0′ →	8444
$\phi =$	35°10.0′ →	8752
$2s =$	140°17.2′	17196
$s =$	70°08.6′ →	46895
$(s - p) =$	− 0°23.6′ →	1
$(s - h_o) =$	35°33.6′	46896
$\phi =$	35°10.0′ (check)	17196
		2)29700
$\frac{1}{2} Z =$	44°44.0′	← 14850
$Z =$	89°28.0′NW	
$Z_n =$	270°32.0′	

FIG. 6.—WORK FORM FOR EXAMPLE 3

The second condition is that in which δ and ϕ are given, and it is required to find Z when the body is at elongation. The most favorable position for accurate azimuth determination occurs when the star appears to be moving vertically for some time. In this position it is at east or west elongation, depending on whether the body is east or west of the observer's meridian. In these positions the PZS triangle becomes a right triangle in which the angle at S (at the star) equals 90°. Hence, at the instant of elongation,

$$\csc Z = \frac{\sec \delta}{\sec \phi} \qquad (10)$$

$$\sec t_Z = \frac{\tan \delta}{\tan \phi} \qquad (11)$$

and

$$\csc h_o = \frac{\csc \phi}{\csc \delta} \qquad (12)$$

Because the cosecant is always greater than unity, Eq. 10 is impossible unless δ is numerically greater than ϕ; and since h_o is always positive, Eq. 12 is impossible unless ϕ and δ have the same sign. Hence, for any latitude, elongation does not occur except for these circumpolar stars whose declination is greater than the observer's latitude and of the same sign. Only δ and ϕ are necessary, hence Eq. 10 alone will give the azimuth when the time of elongation is known. Eq. 11 is useful in predicting the time at which elongation will occur on any given date. Eq. 12 is of no practical use, since azimuth in the region of elongation may remain almost constant for large variations in t_Z and h_o.

The field work for this problem is simple. The transit telescope (vernier reading 0°00′) is directed to the star a few minutes ahead of the predicted time of elongation. With the instrument set in this position, the vertical cross wire is kept centered on the star by the lower clamp tangent screw. When the horizontal motion ceases and the star moves vertically, the telescope is plunged and a point is marked 200 ft or more ahead. This marked line makes an angle Z (given by Eq. 10) with true north. If the star was at east elongation, true north is to the left; if at west elongation, true north is to the right.

TABLE 1.—CHANGES IN ALTITUDE AND AZIMUTH FOR SELECTED STARS AT VARIOUS LATITUDES AND LOCAL HOUR ANGLES (IN DEGREES OF ARC)

Latitude,[a] ϕ	Local hour angle,[b] t_Z	Altitude, h_o	Azimuth angle, Z, from north
(1)	(2)	(3)	(4)
(*a*) Capella, $\delta = +45°57'$[c]			
1	63–76	19.8–10.4	40.9–43.3
5	62–80	22.8–10.5	41.7–44.1
10	59–85	28.5–10.6	42.7–44.8
15	54–90	35.5–10.7	43.6–45.0
20	50–90	41.7–14.2	45.5–45.8
25	47–90	47.2–17.7	48.4–46.8
30	41–90	54.4–21.1	51.7–48.1
35	35–73	61.5–35.4	56.6–54.7
40	28–56	68.1–49.5	64.5–62.4
(*b*) Vega, $\delta = +38°44'$			
1	65–78	20.0–10.0	49.0–51.1
5	60–81	26.4–10.2	49.2–51.7
10	57–85	31.9–10.1	50.6–52.4
15	52–89	38.8–10.0	52.3–52.6
20	46–90	46.4–12.3	54.7–53.2
25	39–90	54.5–15.3	58.1–54.2
30	32–90	62.4–18.1	63.5–55.4
35	29–49	66.6–51.1	72.8–70.1
(*c*) Aldebaran, $\delta = +16°24'$			
1	50–79	38.4–10.8	69.6–73.4
5	42–81	47.4–10.0	71.0–74.1
10	32–82	58.2–10.4	74.8–74.9
15	25–83	65.9–10.7	83.0–75.6
(*d*) Regulus, $\delta = +12°14'$			
1	50–80	39.2–10.0	75.3–78.0
5	36–80	53.7–10.8	76.5–78.7
10	25–81	65.4–10.8	83.0–79.6
15	45–82	36.6–10.7	86.0–80.3

Latitude,[a] ϕ	Local hour angle,[b] t_Z	Altitude, h_o	Azimuth angle, Z, from south
(1)	(2)	(3)	(4)
(*e*) Acrux, $\delta = -62°48'$			
−1	60–70	14.0– 9.8	23.9–25.7
−5	60–77	17.7–10.3	24.4–26.7
−10	55–87	24.3–10.3	24.1–27.4
−15	55–90	28.8–13.5	25.1–27.8
−20	52–90	34.6–17.8	25.8–28.5
−25	53–90	38.6–22.1	27.7–29.3
−30	50–90	44.3–26.5	29.1–30.5
−35	50–90	38.6–30.7	31.7–31.9
−40	47–90	54.1–34.9	34.5–33.6
−45	44–90	59.4–39.3	38.3–35.8
−50	40–75	65.0–49.3	43.6–42.3
−55	55–62	61.6–55.5	51.3–50.0
(*f*) Canopus, $\delta = -52°40'$			
−1	65–74	15.7–10.5	35.0–36.5
−5	61–80	21.3–10.1	34.9–37.5
−10	60–86	26.0–10.3	35.9–38.1
−15	56–90	32.3–11.8	36.7–38.5
−20	54–90	37.4–15.8	38.3–39.2
−25	49–90	44.2–19.6	39.9–40.3
−30	47–90	49.1–23.4	43.0–41.6
−35	45–90	53.9–27.1	46.9–43.2
−40	38–74	61.3–39.7	51.4–49.5
(*g*) Sirius, $\delta = -16°38'$			
−1	50–80	38.4– 9.9	69.6–73.4
−5	44–81	45.4–10.0	71.5–74.1
−10	32–82	58.2–10.4	74.8–74.9
−15	30–83	61.1–10.8	82.9–75.6

[a] Positive latitudes are north; negative, south. [b] East or west of observer's meridian. [c] Positive declinations are north; negative, south.

The third condition occurs when δ, ϕ, and t_Z are given, and it is required to find Z when the body is near elongation. With the increasing facilities of radio, standard time or zone time can be known accurately at any date. Thus, with δ, ϕ, and the local longitude known, one may determine Z independently of h_o and with greater accuracy, if the star is within the region of elongation.

Table 1 gives an approximate range of values of t_Z, h_o, and Z at various latitudes for a few circumpolar stars. Thus, to an observer at latitude 35°N, the value of Z to the star Capella varies from about 56.6° to 54.7°, as t_Z ranges from 35° to 73°, and h_o varies from 61.5° to 35.4°. Hence, within this region a small error in t_Z will cause no appreciable error in Z. It should be noted that the range of values of the local hour angle refers to the period either before or after the transit of the star across the observer's meridian. Hence t_Z is either east or west and the corresponding values of Z are northeast or northwest if the latitude is north, southeast or southwest if the latitude is south.

Application of the Napier rules to each of the right triangles in Fig. 3 gives, after reducing,

$$\tan \phi_F = \tan \delta \sec t_Z \qquad (13)$$

and

$$\cot Z = \frac{\cot t_Z \sec \phi_F}{\csc (\phi_F - \phi)} \qquad (14)$$

(*a*) DETERMINATION OF t_Z

Local Civil Time	=	21 hr 02 min 20 sec 7/1/46
Time Zone	=	6 hr
Greenwich Civil Time	=	3 hr 02 min 20 sec 7/2/46
(From Nautical Almanac)		0°38.5′ 45°37.5′ 0°05.0′
t_G	=	46°21.0′
λ	=	96°20.0′
t_Z	=	− 49°59.0′

(*b*) SOLUTION BY EQS. 13 AND 14

δ = 38°43.9′	→ log tan = 9.90420		
t_Z = 49°59.0′	→ log sec = 0.19178	→ log cot	= 9.92407
ϕ_F = 51°16.8′	← log tan = 0.09598	→ log sec	= 0.20376
ϕ = 32°05.4′			0.12783
$(\phi_F - \phi)$ = 19°11.4′ →	....	log csc	= 0.48320
Z = 66°11.6′NE	....	← log cot	= 9.64463
Z_n = 66°11.6′			

(*c*) ALTERNATE SOLUTION BY EQS. 4, 5, 6, AND 15

t_Z = 49°59.0′ → A 11585			A = log csc × 10^5
δ = 38°43.9′ → B 10786 → A 20365.5			B = log sec × 10^5
γ = ... A 22371 → B 9587	→ B 9587 → A 22371		
ϕ_F[a] = 51°16.8′ .... ← A 10778.5			
ϕ = 32°05.4′			
$(\phi_F - \phi)$ = 19°11.4′ →	B 2483		
h_o = ...	A 12070 → B 18508.5		
Z[b] = 66°11.6′NE	← A 3862.5		
Z_n = 66°11.6′			

[a] ϕ takes the same quadrant as t_Z, and the sign of δ. [b] Z is acute if $(\phi_F - \phi)$ is positive; obtuse if $(\phi_F - \phi)$ is negative.

FIG. 7.—WORK FORM FOR EXAMPLE 4

The angle Z may approach 90°, but never 0°, hence the cotangent was selected instead of the tangent in deriving Eq. 14. For convenience in applying Eqs. 13 and 14, the work form shown in Figs. 7(*a*) and 7(*b*) was devised.

As an alternate solution, Eqs. 4, 5, and 6, and

$$\csc Z = \frac{\csc \gamma}{\sec h_o} \qquad (15)$$

may be used. These equations are derived by applying the Napier rules to Fig. 3. By the aid of the work form shown in Fig. 7(*c*) this set of formulas is adapted to a table of log secants and cosecants.[1a] This alternate solution, although apparently longer, is believed to be much easier than that by Eqs. 13 and 14. Example 4 illustrates both methods.

Example 4.—During the evening of July 1, 1946, a pointing is made to the star, Vega, in the eastern sky. The horizontal motion of the transit is clamped at 21 hr 02 min 20 sec, at the instant the vertical cross hair is on Vega. The Greenwich hour angle of Vega is 46°21.0′, and its declination is 38°43.9′. It is required to calculate the azimuth for an observer at latitude 32°05.4′N, longitude 96°20.0′W by (*a*) Eqs. 13 and 14 and (*b*) Eqs. 4, 5, 6, and 15.

Inasmuch as altitude observations read by the vertical arc of the engineer's transit are scarcely accurate to the nearest minute, it is highly desirable to determine azimuths independently of the altitudes. The second and third of the foregoing cases satisfy this condition and may be classed as favorable methods for accurate azimuth determination.

Most Favorable Conditions for Accuracy

Certain fundamental relations applying to a spherical triangle are:

$$\sin h_o = \sin \delta \sin \phi + \cos \delta \cos \phi \cos t_Z \quad (16)$$

$$\cos h_o \cos Z = \sin \delta \cos \phi - \cos \delta \sin \phi \cos t_Z \quad (17)$$

$$\sin t_Z \cos \delta = \sin Z \cos h_o \quad (18)$$

and

$$\sin \mathrm{ZSP} \cos \delta = \sin Z \cos \phi \quad (19)$$

The law of cosines gives Eq. 16; Eqs. 18 and 19 are applications of the law of sines. Although Eq. 17 is unusual, it is derived elsewhere[2] and will not be derived here.

Latitude.—To evaluate the exact effect on ϕ of any error in t_Z, ϕ may be differentiated with respect to t_Z, assuming h_o and δ to be constant. Accordingly, from Eq. 16,

$$0 = \sin \delta \cos \phi \frac{d\phi}{dt_Z} + \cos \delta \left(- \sin \phi \cos t_Z \frac{d\phi}{dt_Z} - \cos \phi \sin t_Z\right) \quad (20)$$

and

$$\frac{d\phi}{dt_Z} (\sin \delta \cos \phi - \cos \delta \sin \phi \cos t_Z) = \cos \delta \cos \phi \sin t_Z \quad (21)$$

From Eqs. 17, 18, and 21,

$$\frac{d\phi}{dt_Z} = \tan Z \cos \phi \quad (22)$$

This shows that ϕ is entirely independent of t_Z when $Z = 0°$, or when the body is on the observer's meridian. Favorable positions occur when the body is near the meridian—and all the more so for large latitudes.

[1a] Readers who would like a set of these tables to every 0.2′ of arc should correspond with the author at the Univ. of North Carolina, Chapel Hill, N. C.

[2] "Textbook on Practical Astronomy," by George L. Hosmer, John Wiley & Sons, Inc., New York N. Y., 3rd Ed., p. 256.

Referring to Example 1, it follows from Eqs. 18 and 22 that an error of 10′ of angle in t_Z will cause an error in ϕ of only 0.3′ of arc. Differentiating ϕ with respect to h_o while treating t_Z and δ as constant gives, by Eqs. 16 and 17,

$$\frac{d\phi}{dh_o} = \sec Z \quad \text{(23)}$$

which shows that the most favorable position is on or near the meridian, but that an error in h_o causes an equal error in ϕ.

Time or Longitude.—Inversion of Eq. 22 gives:

$$\frac{dt_Z}{d\phi} = \frac{1}{\tan Z \cos \phi} = \frac{\sec \phi}{\tan Z} \quad \text{(24)}$$

which shows that at the instant when $Z = 90°$, $\frac{dt_Z}{d\phi} = 0$; hence, when the body is in that position, an error in ϕ has no effect on t_Z. In other words, the most favorable position of the body is on the prime vertical. It also shows that this statement is all the more true for observers nearer the equator. Thus, for $\phi = 30°$, suppose Z is within 2.5° of the prime vertical; that is, Z_n lies between 87.5° and 92.5°, or between 267.5° and 272.5°. Then $\frac{dt_Z}{d\phi}$ is less than 0.05′. Hence, if the dead reckoning latitude is in error by 20′ the calculated value of t_Z will not be more than 1′ of angle in error. Actually, the latitude should be known within narrow limits if it is determined from a previous or nearly simultaneous sight to a body on or near the meridian.

Longitudes may therefore be determined most exactly when the sun or star is on or near the prime vertical, even though the assumed latitude is somewhat in error.

Differentiating t_Z with respect to h_o while treating ϕ and δ as constant gives, by Eqs. 16 and 18,

$$\frac{dt_Z}{dh_o} = -\csc Z \sec \phi \quad \text{(25)}$$

which indicates that when $Z = 90°$, $\csc Z$ is least and hence $\frac{dt_Z}{dh_o}$ is least for any ϕ, and all the more so when ϕ is small. The most favorable position of the body is therefore on the prime vertical. The negative sign indicates that t_Z decreases as h_o increases, but if Z is obtuse this sign is reversed.

Using the data in Example 2 and applying Eqs. 18 and 24, it follows that $\frac{dt_Z}{d\phi} = 0.02'$, hence an error in ϕ of 1° causes an error in t_Z of only 1′. However, Eq. 25 shows that $\frac{dt_Z}{dh_o} = 1.1$, hence an error in h_o causes about the same error in t_Z.

Digressing for a moment, it will be seen that by inversion of Eq. 25,

$$\frac{dh_o}{dt_Z} = -\frac{1}{\csc Z \sec \phi} \quad \text{(26)}$$

Therefore, the computed value of h is most favorably determined, in so far as an error in t_Z is concerned, when the body is near the meridian, and even more

so for the large latitudes. This is of particular interest in connection with the "line of position" method of determining latitude and longitude.

Azimuth.—From Eq. 24 and a cyclic change of letters,

$$\frac{dZ}{dh_o} = \frac{\sec h_o}{\tan \mathrm{ZSP}} \qquad (27)$$

This shows that an error in h_o causes a negligible effect on Z when the ZSP angle becomes 90° (that is, when the body is at elongation), which cannot be true except when δ is greater than ϕ so that the observed star is circumpolar. It follows that when δ is less than ϕ, or is in the opposite hemisphere, the most

(*a*) Field Notes

Sun Observations—Chapel Hill, N. C.—2/2/46—Clear, cold—T. Rose—Transit No. 6317

Latitude = 35°54.7′N Longitude = 79°03.1′W Time Zone = 5

Telescope	Object	Time	Vertical circle	Horizontal circle	Magnetic needle
Normal	Line	P.M.	....	0°00′	N20°00′W
		2 hr 06 min	32°47.0′	230°36.0′	
		2 hr 10 min	31°54.0′	232°11.0′	
		4 hr 16 min	64°41.0′	462°47.0′	(= Sum)
		2 hr 08 min	32°20.5′	231°23.5′	(= Average)
Reversed		2 hr 14 min	32°02.0′	52°38.0′	
		2 hr 18 min	31°03.0′	54°18.0′	
		4 hr 32 min	63°05.0′	106°56.0′	(= Sum)
		2 hr 16 min	31°32.5′	53°28.0′	
				(232°28.0′)	(= Average)
Average of normal and reversed readings		2 hr 12 min	31°56.5′	232°25.75′	

Eastern Standard Time = 14 hr 20 min
Greenwich Civil Time (GCT) = 19 hr 20 min
Declination for 0 hr GCT 2/2/46 = − 17°02.8′
Correction (19.2 × 0.72) = 13.8′
δ = − 16°49.0′

Observed altitude (uncorrected) = 31°56.5′
Parallax and refraction = − 1.5′
Observed altitude (corrected) = 31°55.0′

(*b*) Work Form

			B
δ =	− 16°49.0′		
p =	106°49.0′		
h_o =	31°55.0′ →	....	7119
ϕ =	35°54.7′ →	....	9156
$2s$ =	174°38.7′		16275 →
s =	87°19.35′ →	....	133055
$(s - p)$ =	− 19°29.65′ →	....	2564
$(s - h_o)$ =	55°24.35′		135619
ϕ =	35°54.7′	(check)	16275 ←
			2)119344
$\frac{1}{2} Z$ =	75°20.4′	....	← 59672
Z =	150°40.8′NW		
Z_n =	209°19.2′		

$$\sec^2 \tfrac{1}{2} Z = \frac{\sec s \sec (s - p)}{\sec h_o \sec \phi}$$

$$s = \tfrac{1}{2}(p + h_o + \phi)$$

$$B = \log \sec \times 10^5$$

Fig. 8.—Example of Azimuth Determination with Sun in Unfavorable Position

favorable position of the body for azimuth determination will depend on both h_o and the ZSP angle. The relative error in Z is smallest when angle ZSP is large and h_o is small.

From Eq. 19 it is seen that the maximum value of angle ZSP occurs simultaneously with the maximum value of Z; that is, when the body is on or near

the prime vertical. This position of the body may or may not be near the horizon; if it is, altitudes should not be less than about 15° because of the uncertainty of horizontal refraction. Using the data in Example 3 and applying Eqs. 19 and 27, it follows that $\frac{dZ}{dh_o} = 0.7'$, and hence an error in h_o of 1′ causes an error in Z of 0.7′.

The field notes and reduction of an actual case of well-executed azimuth determination from sun observations are given in Fig. 8. It will be seen, however, that at this latitude, time, and date, the sun is not in a favorable position for the most accurate results. For greater precision it would have been advisable to take check readings at night on Capella or Polaris.

With Z_n to the sun known, a freehand sketch shows that the true bearing of the line is 232°25.75′ − 209°19.2′ = N23°06.55′W, or its azimuth referred to true north is 336°53.45′. Solving Eq. 19 for angle ZSP,

$$\sin \mathrm{ZSP} = \frac{\sin Z \cos \phi}{\cos \delta} \qquad (28a)$$

or

$$\csc \mathrm{ZSP} = \frac{\csc Z \sec \phi}{\sec \delta} \qquad (28b)$$

Substitution of the foregoing values in Eq. 28*b* gives ZSP = 24°28.6′. Then, from Eq. 27, $\frac{dZ}{dh_o} = 2.6'$. Hence, an error in h_o of, say 1′, would cause an error in Z of 2.6′. Evidently this position of the sun is unfavorable for the most accurate azimuth determination. By interchanging the letters t_Z and Z in Eq. 24, however,

$$\frac{dZ}{d\phi} = \frac{\sec \phi}{\tan t_Z} \qquad (29)$$

which shows that the least error in Z due to an error in ϕ will occur when the body is on the 6-hr circle (t_Z = 90° or 270°).

Combining Eqs. 17 and 18, to eliminate $\cos h_o$, and dividing the resulting expression by $\cos \delta$, gives:

$$\sin t_Z \cos Z = \tan \delta \tan \phi - \sin \phi \cos t_Z \qquad (30)$$

Differentiating Z with respect to t_Z, with δ and ϕ held constant, gives:

$$\frac{dZ}{dt_Z} = \frac{\cot t_Z \cos Z - \sin \phi}{\csc^2 Z} \qquad (31)$$

The least error in Z due to an error in t_Z occurs when the numerator of Eq. 31 equals zero, or when

$$\sin \phi = \cot t_Z \cos Z \qquad (32)$$

Eq. 32 holds true only if angle ZSP equals 90°, or when the body is at elongation. Therefore, the most favorable position of the body for azimuth determination, in so far as a possible error in t_Z (and h_o) is concerned, occurs at or near elongation.

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PAPERS

A CENTROIDAL METHOD OF RIGID-FRAME ANALYSIS

By S. J. Bell,[1] Assoc. M. ASCE

Synopsis

The object of this paper is to present a method for the analysis of rigid frames. Rotations and translations of the ends are represented by a resultant vector at the centroid. By use of this vector and the circle of inertia the amount and direction of the thrust are found. The distance from the centroid to the line of action of the thrust is shown to be a function of the relative rotation of the ends. In a structure with intermediate loads which has been made determinate by release of joints the rotation and the displacement at the centroid determine the thrust required to return it to its original indeterminate position. The moments in the structure are the algebraic sum of the determinate and indeterminate moments. A method of finding the deflection and rotation of any point on the structure is shown. The method is developed analytically and illustrative examples are solved.

1. Introduction

The method of analysis presented in this paper is the result of an effort to shorten the time required for the determination of stresses in vapor lines connecting petroleum refinery fractionating towers to stationary heat exchangers. Application of Castigliano's theorem to the mathematical equations of the shape of the pipe led to the development of an analytical method of solution based on displacements along the principal axes. The solution presented herein is a method of finding the thrust directly from the resultant of these component displacements by use of the circle of inertia.

Some of the problems in which this method will be found useful are the analysis of arches, rigid-frame bridges, pipe expansion bends, rings, box culverts, and other similar structures. It will also be found useful in determining influence lines, secondary stresses, and moment-distribution factors. It is

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **April 1, 1948.**

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applicable to any structure which is subjected to bending stresses and which has no restraint against free flexure except at the two ends.

Notation.—The letter symbols in this paper are defined where they first appear, in the text or by illustration, and are assembled alphabetically in the Appendix for convenience of reference.

2. Moments Produced by End Movements

The displacement of one end of a frame with respect to the other induces a state of internal stress which resists the movement. External forces must be applied to the two ends to maintain equilibrium. In Fig. 1, the forces, F_A and F_B, and the moments, M_A and M_B, are shown applied to ends A and B, which have been displaced with respect to each other. In order to fulfil the laws of statics, F_A must be equal to, parallel to, and opposite in direction to, F_B, and the moment at B must be equal to and opposite in direction to the sum of the moment at A and the moment of F_A about B. By designating the equal forces by F_o and letting them act along a parallel line z-z at a distance Z_A from end A, so that

$$Z_A = \frac{M_A}{F_o} \ldots\ldots (1)$$

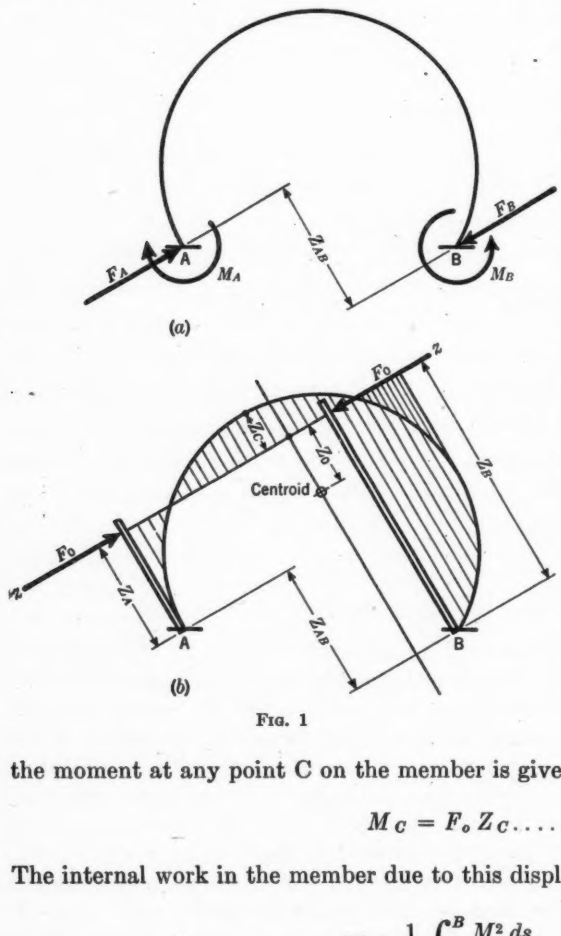

Fig. 1

the moment at any point C on the member is given by

$$M_C = F_o Z_C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The internal work in the member due to this displacement is:

$$W = \frac{1}{2}\int_A^B \frac{M^2\, ds}{E\, I} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which M is the moment; ds is an element of length; E is the modulus of elasticity; and I is the moment of inertia of the cross section. If ϕ is the an-

gular displacement of end A with reference to end B, by Castigliano's theorem,

$$\phi = \frac{\partial W}{\partial M} = \frac{1}{E\,I}\int_A^B M\,ds \qquad (4)$$

Substituting Eq. 2 in Eq. 4,

$$\phi = \frac{F_o}{E\,I}\int_A^B Z\,ds = \frac{F_o\,Z_o\,L_o}{E\,I} \qquad (5)$$

or

$$Z_o = \frac{\phi\,E\,I}{F_o\,L_o} \qquad (6)$$

in which Z_o is the distance from the line of action of F_o to the centroid and L_o is the length of the member.

For $F_o = 0$, that is, where equilibrium is maintained by equal and opposite moments applied at points A and B,

$$\phi = \frac{1}{E\,I}\int_A^B M\,ds = \frac{M\,L_o}{E\,I} \qquad (7)$$

the moment being constant at all points. In this case $Z_o = \infty$.

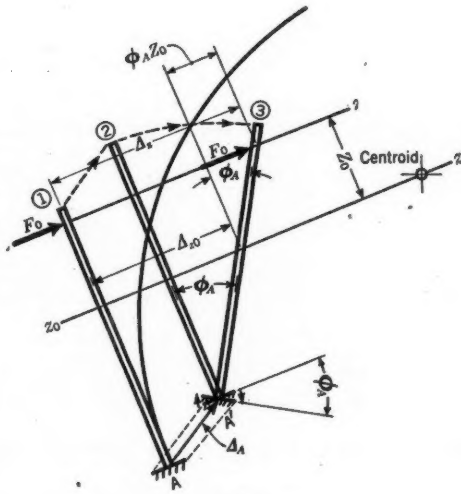

FIG. 2

The expression for internal work may be written:

$$W = \frac{1}{2}\int_A^B \frac{(F_o\,Z)^2\,ds}{E\,I} = \frac{F_o^2}{2\,E\,I}\int_A^B Z^2\,ds \qquad (8)$$

However,

$$\int_A^B Z^2\,ds = I_z = I_{zo} + Z_o^2\,L_o \qquad (9)$$

in which I_z is the moment of inertia of the member about line z–z and I_{zo} is the moment of inertia about an axis through the centroid parallel to z–z.

Therefore, the internal work is:

$$W = \frac{F_o^2}{2\,E\,I}(I_{zo} + Z_o^2\,L_o) \qquad (10)$$

The distance through which the force F_o moves may be found, by Castigliano's theorem, to be:

$$\Delta_{ZFo} = \frac{\partial W}{\partial F_o} = \frac{F_o}{E\,I}(I_{zo} + Z_o^2\,L_o) \qquad (11)$$

Substitution of Eq. 5 in Eq. 11 gives:

$$\Delta_{ZFo} = \frac{F_o\,I_{zo}}{E\,I} + \phi\,Z_o \qquad (12)$$

In Fig. 2, end A is represented as being displaced through a linear distance, Δ_A, and an angle, ϕ_A with respect to end B. Force F_o acts through the distance Δ_{ZF_o} along line z–z against an imaginary lever arm, which has an intercept Δ_{zo} on the centroidal axis parallel to z–z. The following relationship is seen to be true:

$$\Delta_{ZF_o} = \Delta_{zo} + \phi_A Z_o \qquad (13)$$

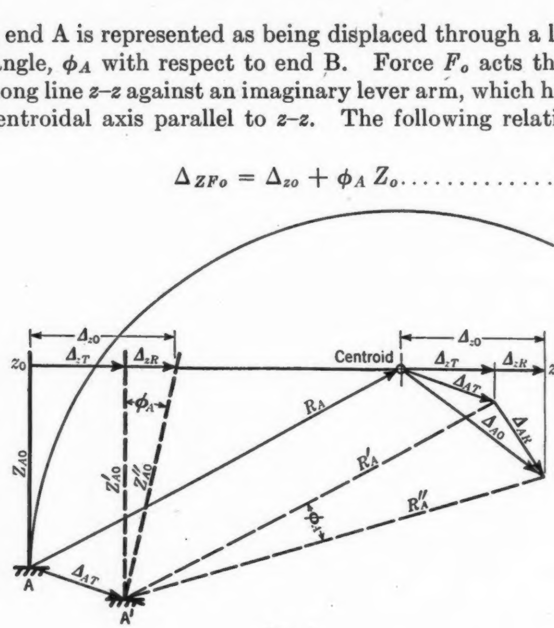

FIG. 3

Substitution of Eq. 13 in Eq. 12 gives:

$$\frac{F_o I_{zo}}{E I} = \Delta_{zo} \qquad (14a)$$

from which

$$F_o = \frac{\Delta_{zo} E I}{I_{zo}} \qquad (14b)$$

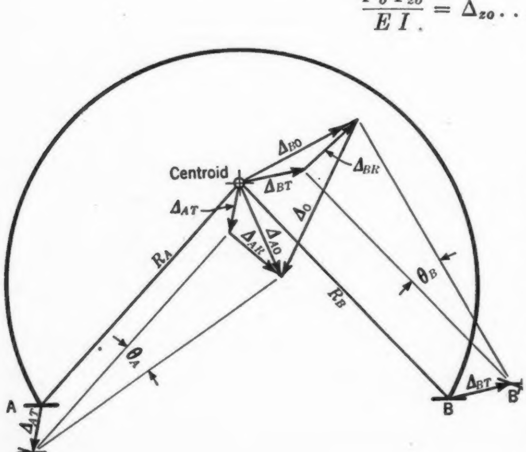

FIG. 4

In Eq. 14*b*, the force F_o is a function of the movement Δ_{zo} (of the imaginary lever arm) along the centroidal axis parallel to the force, as well as of the moment of inertia about this axis. It is independent of the manner of translation and rotation of end A that causes this intercept Δ_{zo}.

In Fig. 3 it is shown that the imaginary lever arm, R_A, may be assumed to extend from end A to the centroid, and the distance Δ_{zo} in Eq. 14*b* is the projection of the movement, Δ_{Ao}, of the end of R_A on the centroidal axis parallel to F_o. This holds true for both ends. If R_A and R_B are lever arms from points A and B to the centroid, both rotation and transla-

tion may be shown as displacements of R_A and R_B away from the centroid. The relative displacement, Δ_o, of end A with respect to end B is the line drawn from the end of R_B to the end of R_A as shown in Fig. 4. The projection of Δ_o on the line of action of F_o is equal to Δ_{zo} as given in Eq. 14b. The projections of Δ_o on the principal centroidal axes (x–x and y–y) give components Δ_{xo} and Δ_{yo}, from which F_x and F_y, the components of F_o parallel to the principal axes may be found by

$$F_x = \frac{\Delta_{xo}\, E\, I}{I_x} \ldots\ldots (15a)$$

$$F_y = \frac{\Delta_{yo}\, E\, I}{I_y} \ldots\ldots (15b)$$

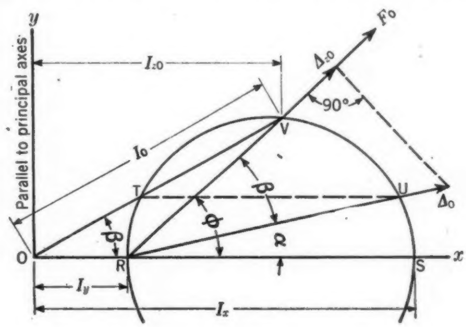

FIG. 5

Knowing the length and direction of Δ_o, the direction and value of the force F_o may be found by use of Mohr's circle as shown in Fig. 5. The construction is as follows:

(1) Draw Δ_o through point R (where OR is equal to I_y if parallel to principal axis x–x and to I_x if parallel to principal axis y–y), cutting the circle at point U.

(2) Through point U draw a line parallel to OR cutting the circle at point T.

(3) Draw a line through points O and T cutting the circle at point V.

(4) A line drawn through points R and V is parallel to F_o.

This may be shown to be true because the slope, ϕ, of F_o with respect to the x–x axis is:

$$\tan \phi = \frac{F_y}{F_x} = \frac{\dfrac{\Delta_y\, E\, I}{I_y}}{\dfrac{\Delta_x\, E\, I}{I_x}} = \frac{\Delta_y\, I_x}{\Delta_x\, I_y} \ldots\ldots\ldots\ldots\ldots\ldots (16a)$$

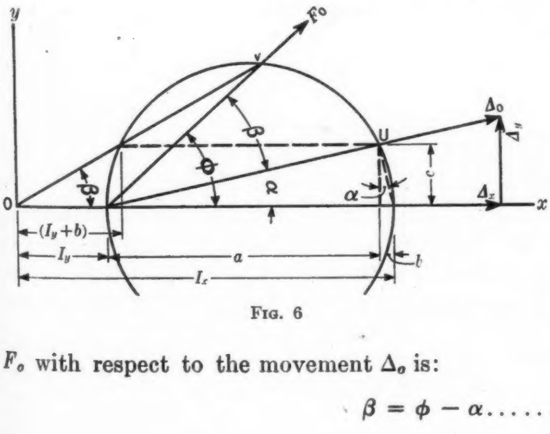

FIG. 6

which, from Fig. 6, will be seen to be:

$$\tan \phi = \frac{c}{a}\frac{I_x}{I_y} \ldots (16b)$$

The slope, α, of Δ_o with respect to the axis x–x is:

$$\tan \alpha = \frac{\Delta_y}{\Delta_x} = \frac{c}{a} \ldots (17)$$

Furthermore, the angle of obliquity, β, of the force F_o with respect to the movement Δ_o is:

$$\beta = \phi - \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18a)$$

and may be found by

$$\tan \beta = \tan (\phi - \alpha) = \frac{\tan \phi - \tan \alpha}{1 + \tan \phi \tan \alpha} \qquad (18b)$$

By substitution of Eqs. 16*b* and 17, as well as linear values from Fig. 6, Eq. 18*b* becomes:

$$\tan \beta = \frac{c(a + b)}{a\, I_y + \frac{c^2}{a} I_x} \qquad (18c)$$

However, by construction—

$$I_x = I_y + a + b \qquad (18d)$$

and

$$\frac{c^2}{a} = b \qquad (18e)$$

Substitution of Eqs. 18*d* and 18*e* in Eq. 18*c* gives:

$$\tan \beta = \frac{c}{I_y + b} \qquad (18f)$$

In Fig. 6, $\frac{c}{I_y + b}$ is the tangent of the angle at the origin, which in turn is equal to the angle of obliquity of force F_o with respect to Δ_o, both of these angles being measured by one half the arc VU.

The value of F_o may be found by use of Eq. 14*b*, Δ_{zo} and I_{zo} being found as shown in Fig. 5. However, the value of F_o may be found without finding Δ_{zo}, the component of Δ_o along the line of action of the force, by

$$F_o = \frac{\Delta_o\, E\, I}{I_o} \qquad (19)$$

in which I_o is a "relative" value of the moment of inertia to be used with the value of the centroidal displacement Δ_o. The value of I_o may be found as shown in Fig. 5.

Eq. 19 may be obtained from Eq. 14*b* by substituting,

$$\cos \beta = \frac{\Delta_{zo}}{\Delta_o} = \frac{I_{zo}}{I_o} \qquad (20a)$$

or

$$\frac{\Delta_o}{I_o} = \frac{\Delta_{zo}}{I_{zo}} \qquad (20b)$$

It should be noted that the equations given in this paper are for structures with constant values of E and I. If either is variable, it must be used in locating the centroid and in finding the moments of inertia. In the frame shown in Fig. 7 the sectional moment of inertia is variable and the structural length must be written:

$$L_o = \int \frac{ds}{I} \qquad (21a)$$

—its dimension being L^{-3}. In like manner, the structural moment of inertia I_z must be written:

$$I_z = \int Z^2 \frac{ds}{I} \qquad \text{(21b)}$$

and its dimension is L^{-1}.

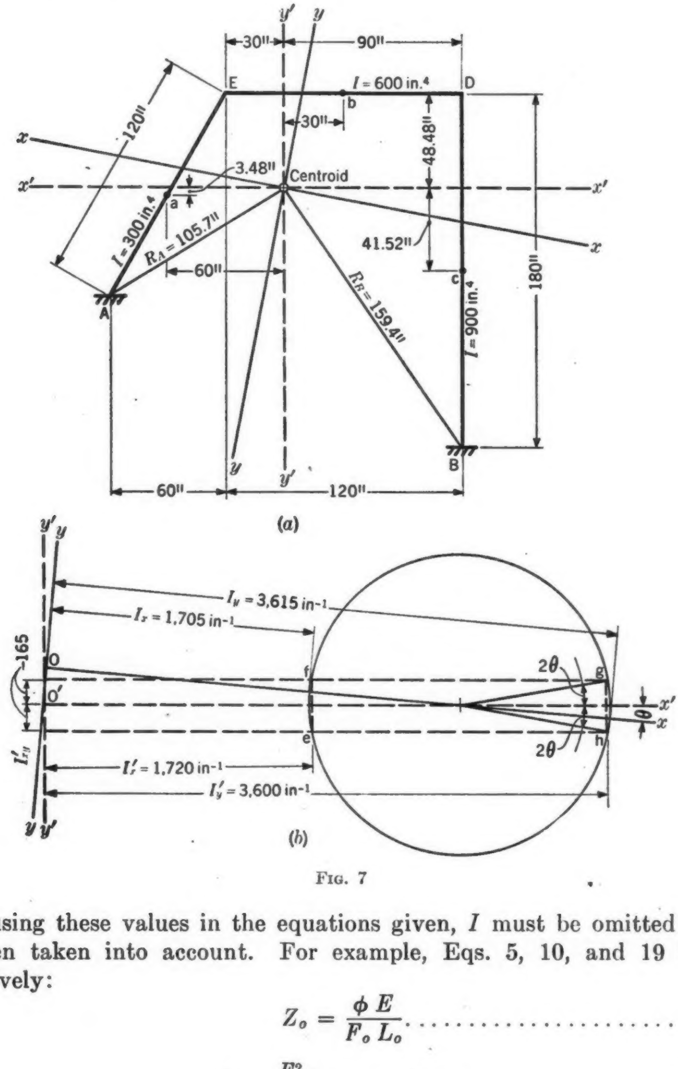

FIG. 7

In using these values in the equations given, I must be omitted since it has been taken into account. For example, Eqs. 5, 10, and 19 become, respectively:

$$Z_o = \frac{\phi\, E}{F_o\, L_o} \qquad \text{(21c)}$$

$$W = \frac{F^2_o}{2\, E} \left(I_{zo} + Z^2_o\, L_o\right) \qquad \text{(21d)}$$

and

$$F_o = \frac{\Delta_o\, E}{I_o} \qquad \text{(21e)}$$

Example 1.—Locate the centroid and the principal axes, and find the principal moments of inertia in the frame shown in Fig. 7. For all members, $E = 30 \times 10^6$. (This example is not an essential part of the method. Its purpose is to find the principal axes and principal moments of inertia to be used in the examples which are to follow.)

(*a*) Location of the Centroid.—Taking moments about the right leg of the bent, $\bar{x} = 90$ in. Taking moments about the top member, $\bar{y} = 48.48$ in.

(*b*) Moments and Products of Inertia About x'–x' and y'–y'.—$I'_x = \frac{1}{12} \times (120 \times 0.866)^2 \times \frac{120}{300} + 0 + \frac{1}{12} \times 180^2 \times \frac{180}{900} + 3.48^2 \times \frac{120}{300} + 48.48^2 \times \frac{120}{600} + 41.52^2 \times \frac{180}{900} = 1{,}720$ in.$^{-1}$ Similarly, $I'_y = 3{,}600$ in.$^{-1}$, and $I'_{xy} = -165$ in.$^{-1}$

(*c*) Location of Principal Axes.—Lay off I'_x and I'_y on a line parallel to one of the axes as shown in Fig. 7. Locate points e, f, g, and h above and below this line a distance equal to I'_{xy}. A circle passed through these points is the circle of inertia. Radii drawn from the center of the circle to points g and h each make an angle equal to $2\,\theta$ with the line. A principal axis bisects one of these two angles, the direction of rotation from the base line to this principal axis being determined by the sign of the criterion, $\frac{I'_{xy}}{I'_x - I'_y}$. A positive sign indicates clockwise rotation and a negative sign, counterclockwise.

By this construction the angle of rotation, θ, is always less than 45°, that is, the axes are always rotated to the nearest principal axes. There is no interchange of the x-axis and y-axis—for example, if I'_y is larger than I'_x, I_y will be the maximum moment of inertia and I_x will be the minimum.

(*d*) Principal Moments of Inertia.—The principal moments of inertia are measured along the base line to the points cut by the circle of inertia. In this example, $I_x = 1{,}705$ in.$^{-1}$ and $I_y = 3{,}615$ in.$^{-1}$

Example 2.—Determine the moments produced in the bent shown in Fig. 7 by the following end movements:

1. End A moves 0.15 in. to the left and up, the direction of movement making the angle $\tan^{-1} 0.4$ with the horizontal.
2. End A rotates counterclockwise through an angle of 6 min.
3. End B moves 0.2 in. down and to the right, making the angle $\tan^{-1} 0.2$ with the vertical.
4. End B rotates clockwise through an angle of 5 min.

(*a*) Direction and Value of Δ_o.—In Fig. 8, using any convenient scale, lay off the linear movements, $\Delta_{AT} = 0.15$ in. and $\Delta_{BT} = 0.2$ in., in their true directions. From the end of Δ_{AT} draw $\Delta_{AR} = R_A\,\theta_A$ perpendicular to R_A and in the direction of the movement of the end of R_A caused by the rotation θ_A. In like manner, draw Δ_{BR} from the end of Δ_{BT}. Vector Δ_o is drawn from the end of Δ_{BR} to the end of Δ_{AR}. Then, since $\theta_A = 6$ min $= 0.00175$ radian, $\Delta_{AR} = 105.7 \times 0.00175 = 0.185$ in., and since $\theta_B = 5$ min $= 0.00145$ radian,

$\Delta_{BR} = 159.4 \times 0.00145 = 0.231$ in. The value of Δ_o is 0.54 in. and the direction is as shown.

(*b*) Line of Action, Direction, and Value of F_o.—Since the base line of the circle of inertia is parallel to the axis x–x, draw a line parallel to Δ_o through the

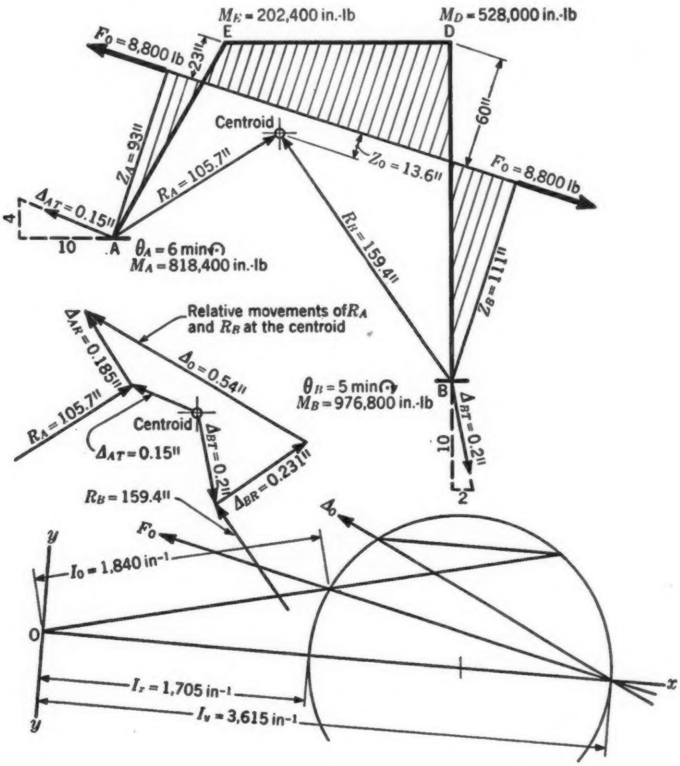

FIG. 8

point where the circle cuts the base line corresponding to I_y. Locate F_o and I_o on the inertia diagram as shown. Then, by Eq. 21*e*, $F_o = \dfrac{0.54 \times 30 \times 10^6}{1{,}840}$ $= 8{,}800$ lb. The distance of the line of action of F_o from the centroid is found by Eq. 5, in which $\phi = (\theta_A - \theta_B) = -0.00175 - 0.00145 = -0.0032$ radian; therefore, $Z_o = -\dfrac{0.0032 \times 30 \times 10^6}{8{,}800 \times 0.8} = -13.64$ in. The negative sign indicates that the moment $F_o Z_o$ of the force acting at the left tends to cause counterclockwise rotation about the centroid. The line of action of F_o serves as a base line for a moment diagram as shown.

3. Application to Loaded Structures

The loaded bent in Fig. 9 is made determinate by releasing end A, leaving end B fixed. The angular change between the ends of any element Δs (as

at C) is:

$$\Delta\phi_C = \frac{M_C\,\Delta s}{E\,I_A}\ldots\ldots\ldots\ldots\ldots\ldots(22)$$

in which M_C is the determinate moment at C. This causes end A to move to A′, a linear distance $\Delta_{AT} = r\,\Delta\phi_C$ and to rotate through the angle $\Delta\phi_C$ with reference to its original position.

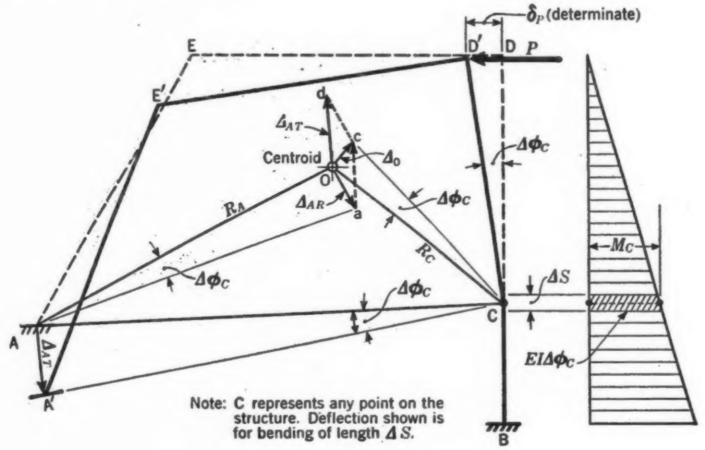

FIG. 9

In order to return this end to its original position it must be moved from A′ to A and rotated through angle $\Delta\phi_C$ in an opposite direction to the first rotation. The movement of the end of R_A due to this translation and rotation is shown by vectors, Δ_{AT} and $\Delta_{AR} = R_A\,\Delta\phi_C$, at the centroid, the resultant of these vectors being Δ_o.

Triangle Oac formed by these vectors is similar to triangle OAC, Oa and ac being perpendicular to, and equal to, $\Delta\phi_C$ times the sides OA and AC, respectively. For this reason Oc is perpendicular to OC and equal to $\Delta\phi_C$ times OC; that is,

$$\Delta_o = R_C\,\Delta\phi_C\ldots\ldots\ldots\ldots\ldots\ldots(23)$$

Thus, the resultant displacement, Δ_o, of the end of the lever arm, R_A, caused by returning end A to its original position, is equal in value and opposite in direction to the displacement of lever arm, R_C, away from the centroid due to the rotation of R_C about point C through the angle $\Delta\phi_C$.

The center of gravity of all the rotations or angular changes is the center of rotation. The total angular change, $\Sigma\,\frac{M\,\Delta s}{E\,I}$, multiplied by the lever arm from this center of rotation to the centroid, will give the resultant movement, Δ_o, of R_A away from the centroid due to bending of all the elements from points A to B. The moment at any point on the fixed bent is equal to the algebraic sum of the determinate moment and the moment set up in the structure as a result of returning ends A and B to their original positions.

Example 3.—Find the moments in the bent shown in Fig. 7 produced by a load of 10,000 lb acting horizontally to the left at the top of the right leg (Fig. 10).

(*a*) Determinate Moments and Displacement Δ_o at the Centroid.—Assume the bent to be made determinate by releasing end A. The total angular change in the determinate structure is $\phi = \frac{1}{2} \times 1{,}800{,}000 \times \dfrac{180.}{900\,E} = 0.006$ radian.

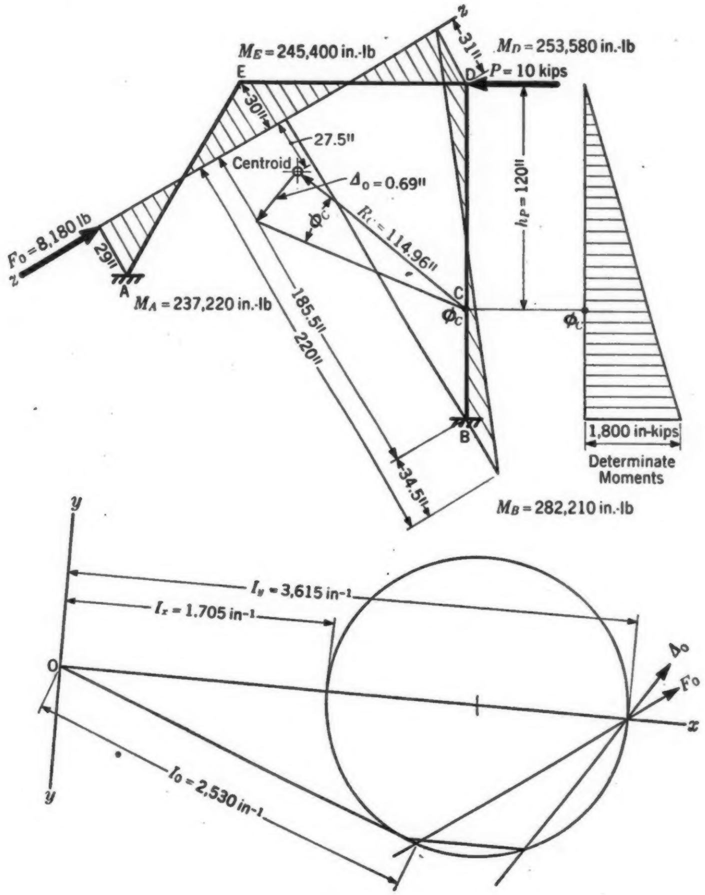

FIG. 10

The center of rotation, C, is on the leg BD at a point corresponding to the center of gravity of the $\dfrac{M}{E\,I}$ -diagram. Its distance from the centroid is $R_C = 114.96$ in. The angular displacement ϕ_C is counterclockwise and causes centroidal displacement $\Delta_o = 114.96 \times 0.006 = 0.69$ in.

(*b*) Indeterminate Moments Produced by Return of the Ends to Their Original Positions.—The return of the ends to their original positions will

cause a relative displacement of R_A away from R_B at the centroid equal and opposite in direction to Δ_o, as previously determined. By use of the circle of inertia, as shown in Fig. 10, the direction of F_o is determined and the value of I_o is found to be 2,530 in.$^{-1}$ Substituting this value in Eq. 19 gives $F_o = \frac{0.69 \times 30 \times 10^6}{2{,}530} = 8{,}180$ lb. Also $Z_o = \frac{180{,}000}{8{,}180 \times 0.8} = 27.5$ in. The rotation of end A in returning to its original position is clockwise (being opposite to the rotation ϕ_C). The force F_o at the left acting on lever arm Z_o will cause a similar rotation about the centroid which determines the direction of Z_o.

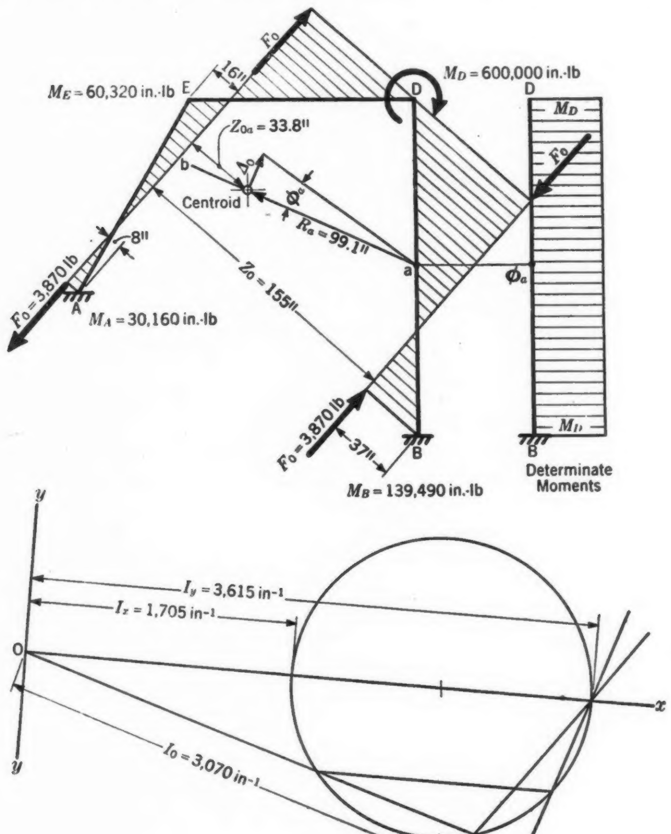

FIG. 11

(c) Final Moment Diagram.—The moment at any point on the bent is equal to the algebraic sum of the determinate and indeterminate moments. Using the line of action of F_o as the base line for plotting both moments, a diagram of the combined moments may be drawn as shown.

4. Moments Produced by Intermediate External Moments

An external moment applied to any point on an indeterminate structure may be assumed to act on either part of the structure as a determinate load. Combination of the resultant moment with the indeterminate moment induced by returning the ends to their original positions will give the actual moment. Solution of a problem of this type is shown in Example 4.

Example 4.—In the bent shown in Fig. 11, the thrust and its lever arm due to an external clockwise moment of 600,000 in.-lb at point D are found to be: $\phi_a = \sum_B^D \frac{M\,\Delta s}{E\,I} = \frac{600{,}000 \times 180}{30 \times 10^6 \times 900} = +\,0.004$ radian; $\Delta_o = \phi_a\,R_a = 0.004 \times 99.1 = 0.396$ in.; $F_o = \frac{\Delta_o\,E}{I_o} = \frac{0.396 \times 30 \times 10^6}{3{,}070} = 3{,}870$ lb; and $Z_{oa} = \frac{\phi_a\,E}{F_o\,L_o} = \frac{0.004 \times 30 \times 10^6}{3{,}870 \times 0.8} = 38.76$ in. By combining the determinate and indeterminate moments a moment diagram may be drawn as shown in Fig. 11.

5. Application to Structures with One End Hinged

The direction and amount of the force F_o acting on the hinged end produced by any linear movement of the hinge with respect to the other end may be found by use of the circle of inertia for the structure at the hinge. The moments and product of inertia for the structure about any pair of rectangular axes through the hinge may be found by transfer from parallel axes through the centroid. The principal moments of inertia are found and the base line is rotated to a position parallel to one of the principal axes at the hinge, in the same manner as previously shown.

The position of the hinge when the structure is in an unstressed condition is treated as the centroid, and the relative displacement of the hinged end away from the end of a rigid lever arm from the other end to this centroid is used as the value and direction of Δ_o to be used in finding F_o. Structures with intermediate loads are treated in the same manner as shown for loaded structures with both ends fixed.

Knowing the direction of F_o and that it acts through the hinge, Z_o (the distance from its line of action to the centroid of the fixed-end structure) is determined. By substitution of this value and the value of F_o in Eq. 5, the rotation of the hinged end with respect to the other end may be found.

Analysis of an unloaded structure for movement of the hinged end is illustrated in Example 5, which is based on Fig. 12. The construction in Fig. 13 shows the analysis of the same structure with an intermediate load and no linear movement of the ends. For loaded structures the rotation of the hinge is the algebraic sum of its rotation for the determinate condition and the rotation due to the action of F_o in bringing the hinged end back to its original position.

Example 5.—Find the moments produced in the bent shown in Fig. 12 by moving end A 1 in. down and to the right at an angle of 45° with the horiozntal.

(*a*) Circle of Inertia at Point A.—From Example 1, the moments of inertia and product of inertia about horizontal and vertical axes are $I_{xo} = 1{,}720$ in.$^{-1}$;

$I_{yo} = 3{,}600$ in.$^{-1}$; and $I_{xyo} = -165$ in.$^{-1}$ The distances of the centroid from the horizontal and vertical axes at point A are $\bar{x} = +90$ in. and $\bar{y} = +55.44$ in. The moments of inertia and product of inertia about parallel axes at point A are: $I'_{xA} = 1{,}720 + 55.44^2 \times 0.8 = 4{,}179$ in.$^{-1}$; and $I'_{yA} = 3{,}600 + 90^2 \times 0.8 = 10{,}080$ in.$^{-1}$; and $I'_{xyA} = -165 + 55.44 \times 90 \times 0.8 = +3{,}827$ in.$^{-1}$. Using these values, the principal moments of inertia and the product

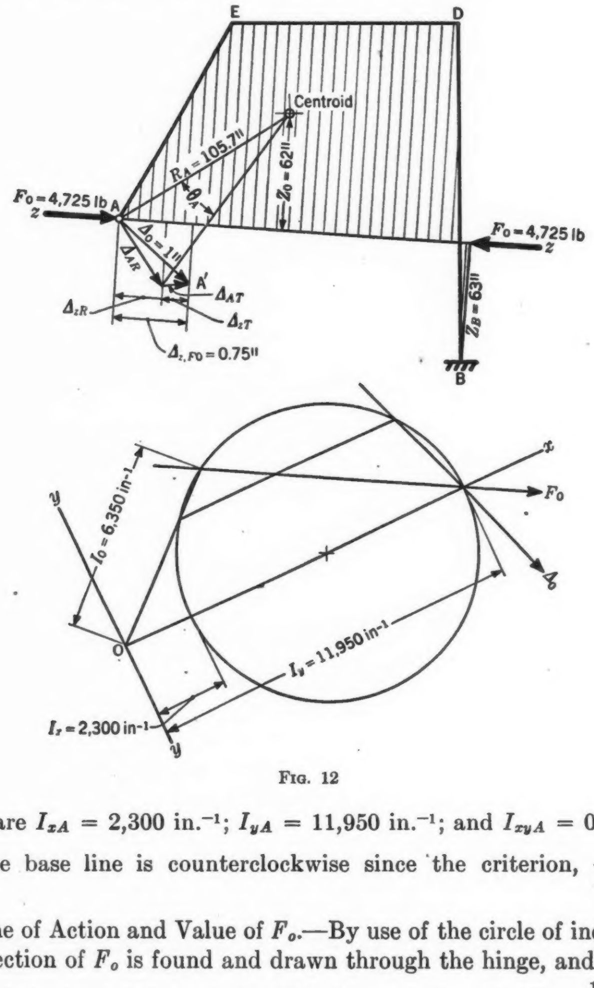

FIG. 12

of inertia are $I_{xA} = 2{,}300$ in.$^{-1}$; $I_{yA} = 11{,}950$ in.$^{-1}$; and $I_{xyA} = 0$. The rotation of the base line is counterclockwise since the criterion, $\dfrac{I'_{xy}}{I'_x - I'_y}$, is negative.

(*b*) *Line of Action and Value of F_o*.—By use of the circle of inertia at point A, the direction of F_o is found and drawn through the hinge, and the value of I_o is found to be 6,350 in.$^{-1}$ Substituting in Eq. 19, $F_o = \dfrac{1 \times 30 \times 10^6}{6{,}350} = 4{,}725$ lb. Knowing the value and line of action of F_o, the moments in the bent can be determined.

(*c*) *Rotation of the Hinged End*.—The value of Z_o is found by construction to be 62 in. Substituting in Eq. 5, $\theta_A = \dfrac{4{,}725 \times 62 \times 0.8}{30 \times 10^6} = 0.0078$ radian.

Rotation θ_A is counterclockwise, since the moment of F_o at the left about the centroid is counterclockwise. The translational and rotational components of the end movement, Δ_o, may be found graphically as shown in Fig. 12.

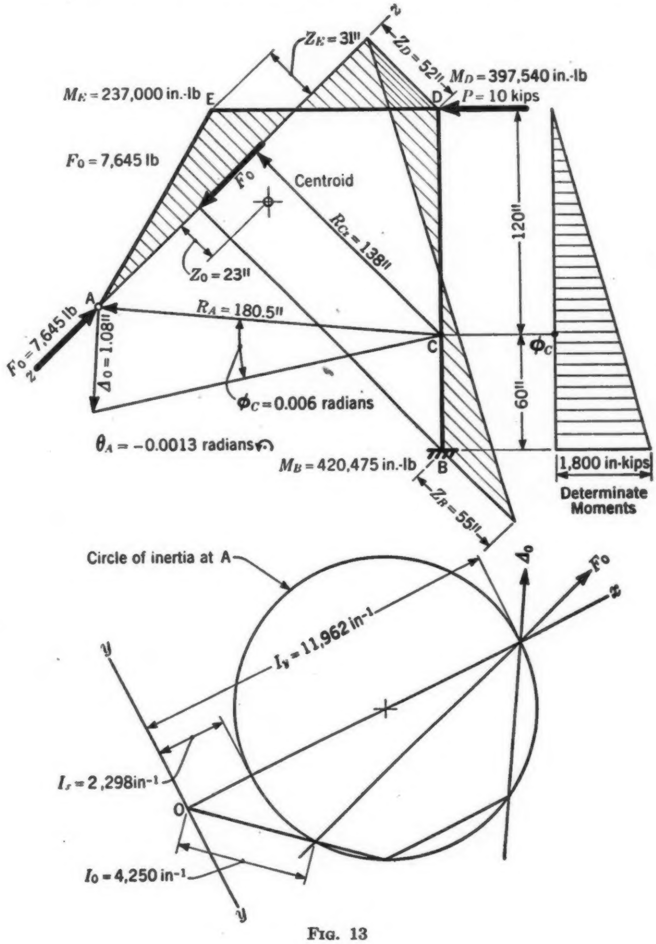

FIG. 13

6. Application to Structures with Both Ends Hinged

For structures with both ends hinged, the line of action of F_o passes through the two hinges. The inertia about this line may be found by transfer from the circle of inertia at the centroid. Using the value thus determined as I_z, F_o may be found by Eq. 11. The relative rotation of the two ends is determined for any value of F_o since Z_o is known by construction.

Part of Δ_z is due to the rotation of the ends about the centroid, the remainder being due to translation without rotation. That is,

$$\Delta_z = \Delta_{zR} + \Delta_{zT} = \frac{F_o\, I_{zo}}{E\, I} + \frac{F_o\, Z^2_o\, L_o}{E\, I} \quad \ldots\ldots\ldots\ldots\ldots (24)$$

from which,

$$\Delta_{zT} = \frac{Z^2{}_o\, L_o}{I_{zo} + Z^2{}_o\, L_o} \times \Delta_z \quad \text{(25a)}$$

and

$$\Delta_{zR} = \frac{I_{zo}}{I_{zo} + Z^2{}_o\, L_o} \times \Delta_z \quad \text{(25b)}$$

Knowing these components of the translational and rotational movements the actual movements of the ends may be found by construction as shown in Fig. 14.

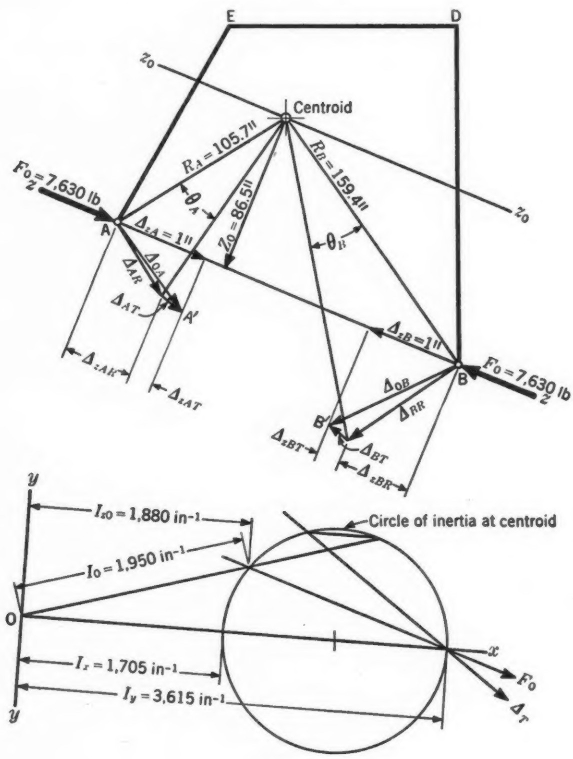

FIG. 14

Example 6.—Find the movements produced in the bent shown in Fig. 14 by moving the hinges 2 in. closer together. Assuming this movement equally divided between the two ends, find the rotation and actual movement of each hinge.

(*a*) Evaluation of I_z and F_o.—The moment of inertia about a line, z_o–z_o, through the centroid parallel to the line of action of F_o is found from the circle of inertia at the centroid to be $I_{zo} = 1,880$ in.$^{-1}$ From construction, $Z_o = 86.5$ in., and the inertia about the axis z–z is $I_z = 7,866$ in.$^{-1}$ Using this value of I_z in Eq. 11, $F_o = 7,630$ lb.

(*b*) Rotations and Movements of the Ends.—Assuming that point A moves 1 in. toward point B, the rotational component of this movement is $\Delta_{zAR} = 0.76$ in., and the translational component is $\Delta_{zAT} = 0.24$ in. Similarly, for a 1-in. movement of point B toward point A, $\Delta_{zBR} = 0.76$ in. and $\Delta_{zBT} = 0.24$ in.

The movement Δ_{AR} is perpendicular to R_A and the movement Δ_{BR} is perpendicular to R_B. Knowing the components of these movements along the axis z–z, the amount of each movement may be found graphically as shown at points A and B in Fig. 14. The direction of the translational movements Δ_{AT} and Δ_{BT} is the same as Δ_T found by the circle of inertia at the centroid. The actual movements of the ends A and B are Δ_{oA} and Δ_{oB}, and the rotations are found to be $\theta_A = 0.0088$ radian, and $\theta_B = 0.0088$ radian, respectively.

Example 7.—Find the moments produced in the bent by the load shown in Fig. 10 if both ends are hinged.

(*a*) Determinate Moments.—Assuming the reaction at point A to act on the hinge in the direction AE and taking moments about points B and D, the determinate reactions are found to be 9,280 lb and 9,660 lb, respectively. The moment diagram for determinate moments is shown in Fig. 15. The centers of rotation for BD and DE are shown at a and b, respectively, and their combined center of rotation is shown at C. The rotation, ϕ_C, which is equal to the sum of the values $\sum \frac{M\,\Delta s}{E\,I}$ for BD and DE, equals 0.0064 radian.

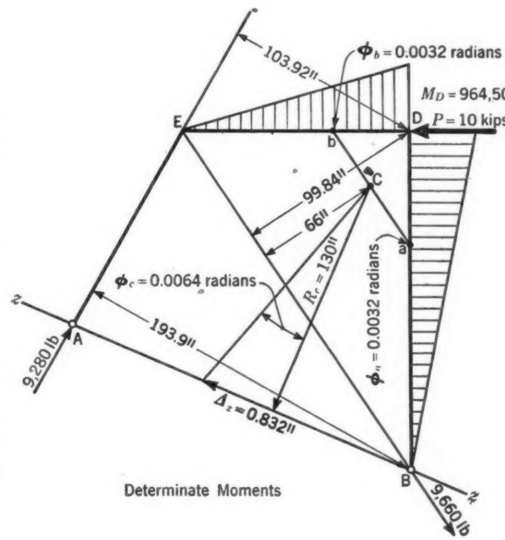

FIG. 15

(*b*) Value of F_o and of Combined Moments and Reactions.—The distance Δ_z through which F_o must act to bring the ends of the deflected determinate structure back to their original positions at points A and B is given by $\Delta_z = 0.0064 \times 130 = 0.832$ in. Substituting this value in Eq. 11, $F_o = 3{,}175$ lb. With F_o known, the diagram of moments may be drawn as shown in Fig. 16, in which the reactions at points A and B are the resultants of the determinate reactions and the forces F_o.

7. Deflections

At the point of application of a load, P, the displacement in the direction of its action is found by application of Castigliano's theorem to be:

$$\delta_p = \frac{\partial W}{\partial P} = \int M \frac{\partial M}{\partial P} \frac{ds}{E\,I} \qquad (26a)$$

A constant unit force ($P = 1$) acting at any point and in the direction of the desired deflection during the loading of a structure gives:

$$\frac{\partial M}{\partial P} = \frac{\partial M}{1} = m \qquad (26b)$$

in which m is the moment produced by the unit force at any point in the unloaded structure. Substituting Eq. 26b in Eq. 26a gives:

$$\delta_p = \int M\, m \frac{ds}{E\, I} \qquad (26c)$$

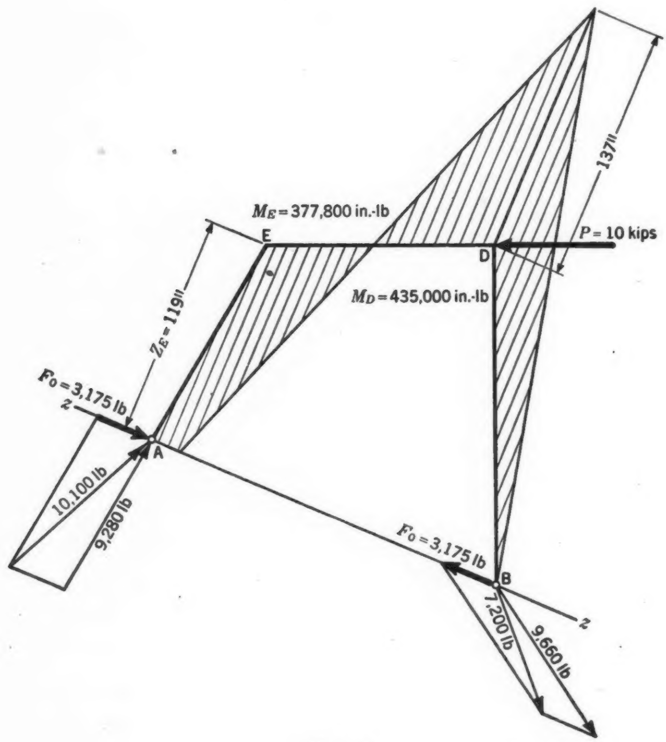

FIG. 16

The moment in a structure caused by movement of its ends is

$$M = F_o\, Z \qquad (26d)$$

and the deflection of the point due to the action of F_o is

$$\delta_p = \int F_o\, Z \frac{m\, ds}{E\, I} = F_o \int Z\, d\phi_1 \qquad (26e)$$

in which $d\phi_1$ is the rotation of any element of length ds caused by moment m. By the assumption that the angular total change in the unloaded structure

caused by the unit force acts at its center of rotation, the deflection becomes:

$$\delta_p = F_o \, Z_1 \, \phi_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

in which Z_1 is the distance from the center of rotation of ϕ_1 to the line of action of F_o.

If m represents the moment at any point due to a unit moment at point p, the rotation at this point may be found, in the same manner, by

$$\alpha_p = F_o \, Z_1 \, \phi_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

in which ϕ_1 is the total angular change in the structure produced by the unit moment and Z_1 is the distance from the center of rotation ϕ_1 to the line of action of F_o. The deflection or rotation found by the foregoing equations is the

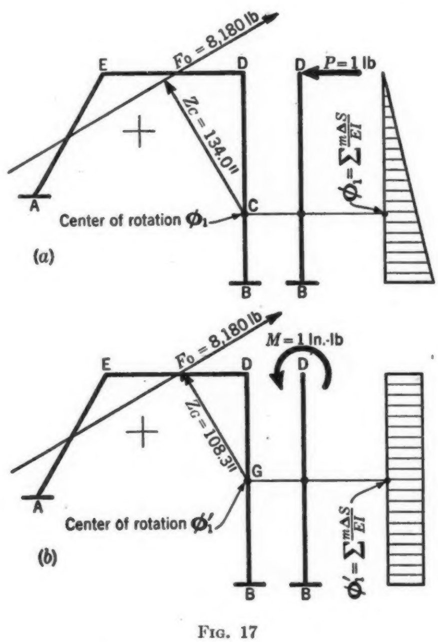

Fig. 17

displacement caused by the action of thrust F_o. This is the total displacement for structures having no intermediate loads. The total displacement for structures with intermediate loads is found by combining the determinate displacement with the indeterminate displacement as previously found.

Example 8.—Find the horizontal deflection and the angular rotation of point D for the loading shown in Fig. 10.

(*a*) Determinate Deflections.—Assume point A free to move and point B to be fixed. From the area of the determinate moment diagram ϕ_C is found to be 0.006 radian and the determinate deflection of point D is $\delta_1 = 0.006 \times 120 = -0.72$ in.

(*b*) Indeterminate Deflections.—With point A free to move and B fixed (see Fig. (17*a*)), a unit horizontal load acting to the left is applied at point D. The center of rotation is at C and $\phi_1 = \frac{1}{2} \times \frac{180 \times 180}{30 \times 10^6 \times 900} = \frac{6}{10^7}$ radian. The distance from the center of rotation to the line of action of F_o is $Z_C = 134.0$ in. and the horizontal deflection of point D caused by the action of F_o is $\delta_2 = 8{,}180 \times 134.0 \times 6 \times 10^{-7} = +\ 0.658$ in.

(*c*) Total Deflection.—The total deflection of point D is $\delta_o = \delta_1 + \delta_2 = -\ 0.062$ in.

(*d*) Determinate Rotation.—The determinate rotation of point D is, as found in Example 8(*a*), $-\ 0.006$ radian.

(*e*) Indeterminate Rotation.—Assuming point A free to move and point B to be fixed in Fig. 17(*b*), the total angular change in DB caused by a unit moment applied at point D and acting on DB is: $\phi'_1 = \Sigma \frac{m\ \Delta s}{E\ I} = \frac{1 \times 180}{30 \times 10^6 \times 900} = \frac{6.667}{10^9}$ radian. The center of rotation is at point G, the midpoint of BD, and its distance from the line of action of F_o is $Z_G = 108.3$ in. The indeterminate rotation is $\alpha_1 = 8{,}180 \times 108.3 \times \frac{6.667}{10^9} = +\ 0.005906$ radian.

(*f*) Total Rotation.—The rotation of D with reference to end B is $\alpha_o = \alpha_2 + \alpha_1 = -\ 0.000094$ radian.

8. Conclusion

The method of solution herein presented offers a simple procedure for the analysis of rigid frames which can be quickly applied, either for the determination of stresses or as a check on other methods. It is offered with the hope that its viewpoint may be of interest and that it may be found useful as a method of analysis.

APPENDIX. NOTATION

The following letter symbols conform essentially to American Standard Letter Symbols for Mechanics, Structural Engineering and Testing Materials (ASA—Z10a—1932) prepared by a Committee of the American Standards Association, with Society representation and approved by the Association in 1932:

C = center of rotation; also, an intermediate point on a structure;
E = modulus of elasticity;
F = force:
  F_A, F_B = force acting at ends A and B, respectively;
  F_o = thrust;

I = moment of inertia:

I_o = a value for I which will give the value of F_o in $F_o = \dfrac{\Delta_o E I}{I_o}$;

I_x, I_y = about the axes x–x and y–y, respectively;

I_z = about z–z, the line of action of F_o;

I_{zo} = about centroidal axis z_o–z_o, parallel to F_o.

L = length: L_o = value of $\displaystyle\int_A^B ds$;

M = moment: M_A, M_B, M_C = at points A, B, and C, respectively;

P = external force;

R_A, R_B = vectors from ends A and B, respectively, to the centroid;

R_C = vector from center of rotation C to centroid;

s = a distance along the center line of the structure;

W = work;

x–x, y–y = rectangular coordinate axes;

Z = a moment arm:

Z_A, Z_C = about points A and C, respectively;

Z_o = of F_o about the centroid;

z–z = line of action of thrust F_o;

z_o–z_o = centroidal axis parallel to the line of action of thrust F_o;

α = angle;

β = angle;

Δ = linear displacement:

Δ_{AR} = linear displacement of end A caused by rotation about the centroid;

Δ_{AT} = linear displacement of end A caused by translation at the centroid;

Δ_o = resultant centroidal displacement;

Δ_z = linear movement of F_o along its line of action;

Δ_{zo} = component of Δ_o along centroidal axis z_o–z_o;

δ = deflection;

θ_A = angular change of end A; and

ϕ = angular change, usually total angular change for part or all of a structure.

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DISCUSSIONS

EXPERIMENTAL OBSERVATIONS ON GROUTING SANDS AND GRAVELS

Discussion

BY ALFRED MACHIS

ALFRED MACHIS,[21] JUN. ASCE.—The author is indebted to those who have taken the time to critically analyze and discuss this paper. He believes that the contribution made by Frederick J. Smith, Jun. ASCE, is especially appropriate, since it serves to focus the attention of the civil engineer on advances made in an allied field relating to a problem very similar to his own.

Conservation of a natural resource of such immense value as ground waters should be of primary importance to the engineer. Large investments are made annually in the construction of water wells and plants to treat the waters obtained from these wells. Experiences in the oil industry indicate that the leakage of an undesirable fluid down the outside of a well casing may be more economically controlled by grouting than by depending upon the separation of the fluids in refining. Similarly, it may be more economical to prevent the leakage of mineral-bearing waters into fresh-water aquifers by grouting than it would be to sustain the economic losses necessitated through the use of mineral-bearing waters.

The large number of successful results secured in the oil well industry indicates that the problem of grouting outside a well casing is not impractical. The author's attention was first directed to this problem of water contamination by an unsuccessful attempt to prevent the leakage of salt-bearing waters down the outside of a well casing into a fresh-water aquifer. Grouting was performed without a thorough understanding of the factors involved. The experimental data of this study were gathered to help provide the engineer with a basis upon which to form a logical solution to the problem.

NOTE.—This paper by Alfred Machis was published in November, 1946, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1947, by James B. Hays; March, 1947, by William S. Foster and June, 1947, by Frederick J. Smith.

[21] Research Associate in Eng., The Johns Hopkins Univ., Baltimore, Md.

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DISCUSSIONS

FORECASTING PRODUCTIVITY OF IRRIGABLE LANDS

Discussion

BY HARRY F. BLANEY

HARRY F. BLANEY,[14] M. ASCE.—For many years the writer has been interested in the relationships between temperature, consumptive use of water, and crop yield. The late Mr. Muldrow is to be complimented on his ingenious method of estimating the productivity of lands on a proposed irrigation project, and his paper should encourage further research on the subject. However, the writer does not agree with some of the statements and results presented.

The author states (under the heading, "Graph") that he was

> "* * * quite surprised by the record in the Imperial Valley in California, where alfalfa grows every month in the year and the farmers harvest eight cuttings; yet the census shows that Imperial County had 114,146 acres of alfalfa in 1939 and that the average yield was 2.99 tons per acre."

The writer does not question the census data, but they probably do not reflect the tremendous amounts of green alfalfa produced and fed to livestock shipped into Imperial Valley from other areas. The Imperial County Agricultural Commissioner reports that in 1939 approximately 75,000 head of cattle and 120,000 head of sheep were fattened. Table 2 is taken from records of the annual crop and livestock reports of the Imperial Irrigation District.

TABLE 2.—HEAD OF LIVESTOCK FED IN IMPERIAL VALLEY

Year	Cattle	Sheep	Dairy	Hogs
1942	84,420	81,080	15,200	42,010
1943	75,260	81,620	13,050	39,005
1944	77,200	113,700	12,776	19,817
1945	116,730	140,593	11,355	18,679

During the winter months livestock are brought into the valley, fed green alfalfa, and are then shipped out to market. The same fields which produce the baled hay also provide the winter pasturage. There is very little permanent pasture in Imperial Valley. In other words, the census data more

NOTE.—This paper by the late W. C. Muldrow was published in February, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1947, by Arthur F. Johnson, H. W. Van Loo, and Will H. Noble; June, 1947, by Charles Kirby Fox; and October, 1947, by M. R. Lewis.

[14] Senior Irrig. Engr., Div. of Irrig., S.C.S., Research, U. S. Dept. of. Agriculture, Los Angeles, Calif.

nearly reveal the amounts of alfalfa harvested and sold, rather than produced; also, the actual yield of alfalfa in the valley is higher than 2.99 tons per acre.

A report on cost of production studies, conducted in 1943 on several representative farms by the Agricultural Extension Service of the University of California and the United States Department of Agriculture, in cooperation with alfalfa growers in Imperial County, states:

> "The average yield for all records including that utilized as pasture was 4.64 tons per acre, ranging from 3.5 to 6.21. Average cost per acre was $72.75; income $104.17 and the net profit $31.42."[15]

Mr. Muldrow's statement that "Imperial Valley does not grow alfalfa to make money * * *" may have been true some years ago. However, in recent years the value of Imperial Valley's alfalfa crop was seldom exceeded by that of any other single crop produced in the valley. For example, the Imperial Irrigation District reports that, in 1943, 114,085 acres produced 16,585 carloads of alfalfa valued at $5,970,600; and, in 1945, when the price of alfalfa was good and hay was scarce, 158,713 acres produced 34,343 carloads valued at $11,848,335. Owners are leasing land suitable for growing alfalfa for more than $40.00 per acre per yr.

TABLE 3.—Consumptive Use of Water by Alfalfa in Irrigated Areas

Location (1)	Mean annual temperature (degrees F) (2)	Season (3)	Consumptive use[a] (ft) (4)
Spokane, Wash.	48	Apr. 12–Oct. 13	2.5
Walla Walla, Wash.	53	Mar. 31–Nov. 5	3.1
American Falls, Idaho	46	May 26–Sep. 16	1.7
Boise, Idaho	50	May 1–Oct. 5	2.5
Cheyenne, Wyo.	45	May 15–Oct. 2	1.9
Worland, Wyo.	44	May 10–Sep. 27	2.1
Missoula, Mont.	45	May 18–Sep. 23	1.8
Logan, Utah	48	May 7–Oct. 11	2.3
Saint George, Utah	....	Apr. 10–Oct. 23	3.1
Sacramento, Calif.	60	Apr. 1–Oct. 31	3.1
Bakersfield, Calif.	65	Apr. 1–Oct. 31	3.3
Antelope Valley, Calif.	62	Apr. 1–Oct. 31	3.2
Imperial Valley, Calif.	73	Jan. 1–Nov. 30[b]	3.7
Phoenix, Ariz.	72	Mar. 1–Nov. 30[b]	3.7
Mesilla, New Mex.	60	Apr. 1–Oct. 31	3.2
Carlsbad, New Mex.	63	Apr. 1–Oct. 31	3.2
Lubbock, Tex.	60	Apr. 1–Oct. 31	3.2
Fort Stockton, Tex.	65	Apr. 1–Oct. 31	3.3
Altus, Okla.	62	Apr. 1–Oct. 31	3.3

[a] Computed from mean monthly temperatures (t), per cent of daytime hours (p), and consumptive-use coefficient (k); consumptive use $= t\,p\,k$. [b] Month of July excluded for rest period.

As indicated by the author, the "consumptive-use-of-water" curve in Fig. 1 represents average valley crops during the growing season. Studies made by the writer in the Upper Rio Grande Basin in New Mexico[16] and in other areas throughout the west, indicate that consumptive use of water shown at the upper end of this curve is probably more representative of the annual consumptive use by alfalfa. For instance, the annual consumptive use of water by alfalfa in Mesilla Valley in New Mexico[16] has been estimated at 4.0 ft, whereas the average annual valley consumptive use is only 2.8 ft. Alfalfa uses more water than other crops. The consumptive use of water by

[15] "1943 Alfalfa Production Cost and Efficiency Analysis, Imperial County, Calif.," Agri. Extension Service, Univ. of Calif., Berkeley, and U. S. D. A., cooperating with alfalfa growers, Imperial County, Calif. (mimeographed).

[16] "Water Utilization, Upper Rio Grande Basin, Part III," by Harry F. Blaney, P. A. Ewing, O. W. Israelsen, C. Rohwer, and F. C. Scobey, National Resources Committee, Washington, D. C., February, 1938.

potatoes should not exceed 2 ft. Table 3 has been prepared to show seasonal consumptive use by alfalfa in various areas of the west. Briefly, the procedure[17] is to correlate existing consumptive use data with monthly temperature, per cent of daytime hours, and growing season. The coefficients so developed for different crops are used to transfer consumptive use data from one section to other areas where only climatological data are available.

The curve "Tons per acre, alfalfa hay = 2,400 day degrees per ton" (Fig. 1) is probably incorrect inasmuch as the author did not have all the facts. For instance, the alfalfa yield data are county averages, whereas temperatures may not represent the area where most of the crop is grown. Most of the alfalfa in Los Angeles County is grown in Antelope Valley and other interior valleys, where the temperatures are similar to those for Bakersfield and Riverside, Calif., rather than for those of Los Angeles. The available heat units in Antelope Valley are about the same as in Bakersfield, and are somewhat higher than those in Los Angeles. If the yield of 5.5 tons of alfalfa per acre, shown for Los Angeles, is moved to the left in Fig. 1 to about 16,500 available degree days, and the yield for Imperial Valley is increased from 3.0 to 4.5 tons per alfalfa, the writer believes the resulting curve would more nearly represent field conditions.

The writer agrees with the thoughts expressed in the "Summary." It is a challenge to both engineers and research workers to improve procedure for forecasting the productivity of irrigable lands and the feasibility of proposed irrigation projects.

[17] "A Method of Estimating Water Requirements in Irrigated Areas from Climatological Data," by Harry F. Blaney and Wayne D. Criddle, SCS, U. S. D. A., June, 1945 (mimeographed).

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DISCUSSIONS

BEAM DEFLECTIONS BY SECOND AND THIRD MOMENTS

Discussion

BY WILLIAM A. CONWELL

WILLIAM A. CONWELL,[12] M. ASCE.—The scholarly presentation of this paper and its adherence to fundamentals in the approach to the solution of deflection problems make it a definite contribution to engineering literature.

In the "Synopsis," the author states "* * * the aim of this paper is to suggest a more direct solution" and further indicates that it is proposed to solve deflection problems without use of the bending moment. It should be noted that, although the author achieves the elimination of the bending moment, he does not do it without introducing another moment concept—namely, that of second and third moments. There is some question as to whether this substitution is an improvement. If the sign convention is remembered, the second and third moments of a force are readily understood. However, the second and third moments of a moment are not so clearly visualized, inasmuch as they involve a coefficient which has no physical significance and so introduces and element of artificiality. Likewise, the expressions for angle changes and deflections (Eqs. 27 and 30) involve divisors which have no direct physical significance and must, therefore, be remembered if used. Furthermore, if the design computations for the structure are available, the bending moments on which to base deflection calculations are also at hand. In most instances, it would be necessary to compute second and third moments. Still further, the bending moment is intimately associated with the geometry of deflections and thus is an invaluable aid in the visualization of structural action.

Since the method of this paper has the effect of separating the mind of the designer from the action of the structure, it should offer something to balance this disadvantage. In the use of the column analogy,[13] for instance, the designer's mind is taken up with the computation of stresses in a fictitious column cross section whose stresses are analogous to the bending moments in the struc-

NOTE.—This paper by Hsu Shih-Chang was published in March, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1947, by A. Floris, Robert B. B. Moorman, and Frank J. McCormick.

[12] Gen. Engr., Structural Eng. and Design Dept., Duquesne Light Co., Pittsburgh, Pa.

[13] "The Column Analogy," by Hardy Cross, *Bulletin No. 215*, Eng. Experiment Station, Univ. of Illinois, Urbana, 1930.

ture being analyzed. In the process of computing column stresses, the designer is not thinking in terms of the action of the structure, but at the conclusion of this work he has a good basis for visualizing it. The only excuse for interposing the column analogy in the analysis is that it produces results more quickly in some cases than would otherwise be obtained and, thus, conserves the time of the designer. Whether the same can be stated of the method under discussion will be determined as it is used in competition with other methods.

Practically the entire portent of this paper is carried in theorems VI and VII. Theorems I to V, inclusive, are almost exclusively devoted to definitions. This raises the question as to whether greater emphasis should not be given to theorems VI and VII than is accorded them in the paper at the end of the series of definitive theorems.

From the beginning of his studies in strength of materials, the engineer has been aware that angle changes vary as $\frac{M\,a}{E\,I}$ and deflections vary as $\frac{M\,a^2}{E\,I}$. Accordingly, since M is generally equal to $\Sigma P\,a$, he concluded at an early point in his progress that angle changes vary as $\Sigma\frac{P\,a^2}{E\,I}$ and deflections vary as $\Sigma\frac{P\,a^3}{E\,I}$. It has remained, however, for the author of this paper to employ this general conclusion in such a way as to produce quantitative results. It is, therefore, possible that the method of second and third moments, when used in conjunction with methods more generally employed, will prove to be a valuable tool in determining deflections.

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DISCUSSIONS

CONTINUOUS FRAME ANALYSIS BY ELASTIC SUPPORT ACTION

Discussion

BY LEROY A. BEAUFOY, A. A. EREMIN, ROBERT B. B. MOORMAN, EDUARDO AGRAMONTE, STEPHEN J. FRAENKEL AND ROBERT L. JANES, AND PHIL M. FERGUSON

LEROY A. BEAUFOY,[28] ESQ.—The method offered by Professors Rathbun and Cunningham is presented in three divisions in terms of its application to different structural types:

(a) Structures in which fixed joints need not be introduced, such as continuous beams on unyielding supports, or single story frames of a type in which only a single sidesway may occur;

(b) Vierendeel trusses; and

(c) Building frames.

The writer will direct his discussion at the same three divisions.

In respect of the first, attention is drawn to certain similarities and differences as compared with an earlier paper by Otto Gottschalk.[29] It is easy to show that some of the authors' expressions are, in fact, the same as expressions derived (from a fundamentally different point of view) by Mr. Gottschalk. Thus, consider a beam AB, fixed at end A and connected at joint B to one or more members (Fig. 14). In adapting the Gottschalk symbols to the authors' notation let S_A be the relative stiffness value for end A; S_B be the sum of the relative stiffness values of all members at end A except member

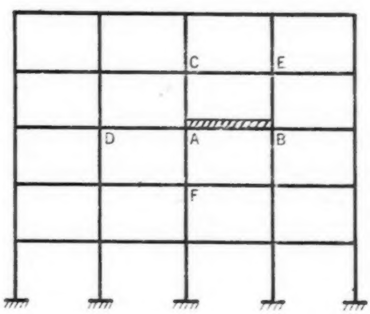


FIG. 14

NOTE.—This paper by J. Charles Rathbun and C. W. Cunningham was published in April, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1947, by L. J. Mensch, Frederick S. Merritt, I. Oesterblom, Thomas C. Kavanagh, A. Floris, and Tao King.

[28] Univ. Reader in Civ. Eng., King's College, Univ. of London, London, England.

[29] "Structural Analysis Based Upon Principles Pertaining to Unloaded Models," by Otto Gottschalk, *Transactions*, ASCE, Vol. 103, 1938, p. 1019.

BA; $-m$ be the rotation at joint B induced by unit rotation at end A; k be the stiffness ($E\,I/L$) of a member; and c be the carry-over factor from point A to point B. Then, among the expressions derived by Mr. Gottschalk are:

$$m = \frac{0.5\,k}{S_B + k} \qquad (49a)$$

$$S_A = (1 - 0.5\,m)\,k \qquad (49b)$$

and

$$c = \frac{1 - 2\,m}{2 - m} \qquad (49c)$$

In Eq. 49a, writing for k its equivalent $\frac{E\,I}{L}$:

$$m = \frac{1}{2\,\frac{S_B\,L}{E\,I} + 2} \qquad (50a)$$

and substituting this in Eq. 49c:

$$c = \frac{1}{2}\left[\frac{L}{L + \frac{3\,E\,I}{4\,S_B}}\right] \qquad (50b)$$

Transposing to the authors' symbols through their definition for Z_B in Eq. 6, that is,

$$Z_B = \frac{1}{4\,S_B} \qquad (51a)$$

—and substituting in Eq. 50b—

$$c = \frac{1}{2} \times \frac{L}{L + 3\,E\,I\,Z_B} = \frac{1}{2} \times \frac{L}{L_{AB}} \qquad (51b)$$

which is the same as the corresponding quantity in Eq. 7.

Again, by substituting for m from Eq. 50a, and writing $E\,I/L$ for k, Eq. 49b can be rewritten as

$$S_A = \frac{4\,\frac{S_B\,L}{E\,I} + 3}{4\,\frac{S_B\,L}{E\,I} + 4}\,\frac{E\,I}{L} \qquad (52a)$$

Substituting $Z_A = \frac{1}{4\,S_A}$ and $Z_B = \frac{1}{4\,S_B}$:

$$Z_A = \frac{L}{3\,E\,I}\left[\frac{(3\,E\,I\,Z_B + L) - \frac{L}{4}}{L + 3\,E\,I\,Z_B}\right] = \frac{L}{3\,E\,I}\left[\frac{L_{AB} - \frac{L}{4}}{L_{AB}}\right] \qquad (52b)$$

which is the same as Eq. 11.

However, although the expressions derived by Professors Rathbun and Cunningham are basically the same as those derived by Mr. Gottschalk, the

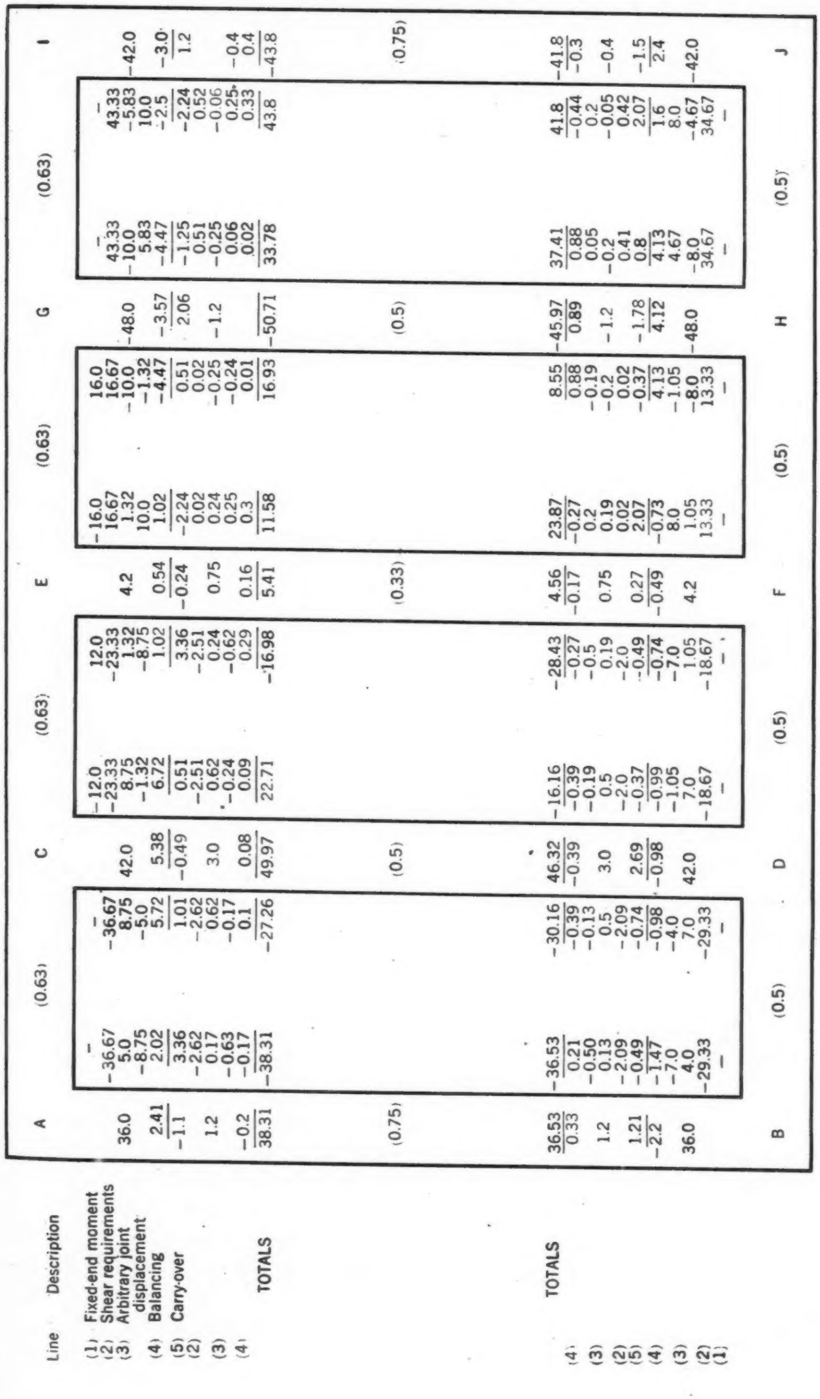


FIG. 15.—COMPLETE SOLUTION OF EXAMPLE 6 BY MOMENT DISTRIBUTION

procedure recommended by the authors is quite different. They aim at obtaining a direct solution for a given case of loading, whereas Mr. Gottschalk, by considering the unloaded structure as an elastic body and determining its shape when deformed in a particular manner, obtains influence lines from which stress values for any loading condition can be obtained. Mr. Gottschalk's method, however, can be used for direct stress analysis. The labor involved in the authors' method is no less than that employed in Mr. Gottschalk's method, and the results are tied to a particular loading instead of being readily applied to any loading. In comparing the two methods, therefore, advantage would appear to be with the earlier one.

In fact, taking Example 3 as an instance and comparing the authors' solution with that by the Gottschalk method, there is added labor in the former because of the necessity for calculating the end moments when the ends are elastically supported (what one might call "elastic end moments"); whereas Mr. Gottschalk uses the more easily determined fixed-end moments. Subsequent processes in both methods are comparable and involve approximately the same amount of labor.

The second application, to Vierendeel trusses, is regarded as quite unsatisfactory. As presented, the method involves so many operations that it will not appeal to many engineers who, by using other methods, can much more easily obtain solutions to structures of the types dealt with. Compare the authors' solution to Example 6, which occupied nearly ten pages of the paper, with Fig. 15, which gives, on a single page, the complete solution by moment distribution, using the device of arbitrary joint displacements.[30] It is recognized that the paper sets out specifically to give an "exact" method, but it is submitted that such a method should offer some advantage over existing methods. Whatever the facts may be as regards the claim that the number of simultaneous equations is largely reduced as compared with the classical methods, it remains true to say that many simultaneous equations are left. For instance, Example 6 involves three sets of two simultaneous equations plus one set of four, and it should be noted that for the solution of these equations an ordinary slide rule is inadequate. As the method does not appear to be adapted to the determination of influence lines, separate solutions involving much the same labor would presumably be needed for each case of loading, whenever more than one loading condition has to be investigated. It must also be emphasized that in the case of a Vierendeel truss with non-parallel chords any method utilizing the shearing forces in the panels will encounter special difficulties because the inclined chord members participate in the resistance to shearing force. It is not considered, therefore, that the authors' claim (that their method is particularly adapted to the solution of Vierendeel trusses) is borne out by the contents of their paper.

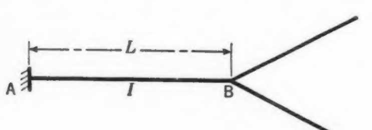


FIG. 16

[30] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, John Wiley & Sons, Inc., New York and London, 1932, p. 228.

The application of the authors' method to building frames may be justified only from the point of view of obtaining an exact solution, for which purpose it has certain merits. Otherwise, it will be found accurate enough in dealing, for example, with the problem in Fig. 16, to start by determining the stiffnesses of the ends AC, AD, and AF, as if all members at points C, D, and F (other than CA, DA, and FA) have their far ends either hinged or fixed. The actual stiffnesses and carry-over factors are determined accordingly, and a solution is readily obtained by the Gottschalk method. It would be easier to assume the far ends to be hinged rather than fixed, since there would then be no unbalanced moments there. The differences between the actual moment and the moments obtained by either of these assumptions will be slight in any case. It may be added that an even quicker solution can be obtained using moment distribution and speeding up the convergence by suitable arbitrary joint displacements.

A final comment relates to a detail in the paper. In Section 7, step (*g*) would be clarified, it is suggested, by adding "the fixed joints must later be released and the moments corrected." In fact the authors do this in Example 6, step (*g*), and mention it again in Example 7, step (*g*). Incidentally, this process alone involves the solution of *a*-groups of *b*-equations each, in which *a* denotes the number of fixed joints and *b* the number of shear displacements.

A. A. Eremin,[31] Assoc. M. ASCE.—A direct algebraic distribution of moments in continuous frames is shown in this paper by solving the illustrative examples. However, distribution of moments may also be clarified by the graphical construction of moments.

The moments in Example 1(*a*), determined by the graphics, are shown in Fig. 17. The distance to the inflection point, a, is

$$a = \frac{e\,L}{e + e_0\,Z_A\,\dfrac{I}{L}} \quad \ldots\ldots (53)$$

and the elastic factor is

$$Z_B = \left(e - e_0\,\frac{a}{L - a}\right)\frac{L}{I} \quad \ldots (54)$$

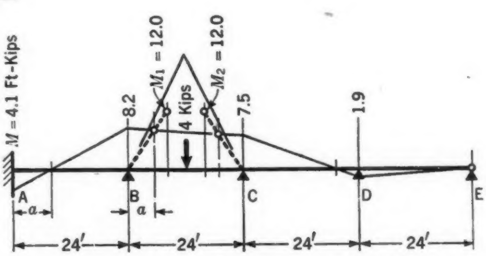


Fig. 17.—Graphical Determination of Moments for Example 1(*a*)

The constants e and e_0 in Eqs. 53 and 54 vary with the shape of the member and may be read from the diagrams prepared by the writer.[32] In a member with a constant section, $e = 0.333$ and $e_0 = 0.167$.

The moment closing line in span BC is constructed through the intersection points on the inflection point verticals cut by lines drawn through the central

[31] Associate Bridge Engr., Bridge Dept., Div. of Highways, State Dept. of Public Works, Sacramento, Calif.

[32] "Analysis of Continuous Frames by Graphical Distribution of Moments," by A. A. Eremin, Sacramento, Calif., 1943.

moments M_1 and M_2, as shown in Fig. 17. Similarity of moment distribution shown in Table 1 and that shown by the graphics in Fig. 17 is obvious.

Final moments produced by a combined loading have been computed by the authors by superposition of moments in Tables 2 and 5. The summation operation may be eliminated by application of the "cross lines." The moments determined by the "cross lines" for Example 3 with no sidesway are shown in Fig. 18. Final moments determined by the graphics in Figs. 17 and 18 differ by less than 2% from those computed by the authors.

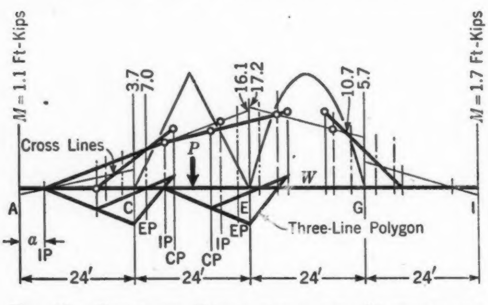


FIG. 18.—GRAPHICAL DETERMINATION OF MOMENTS FOR EXAMPLE 3 WITH NO SIDESWAY

By applying Eqs. 53 and 54 to the authors' method it may be extended to continuous frames composed of members with varying sections. Application of the "cross lines" and graphical construction of sidesway effect are shown elsewhere.[33,34]

It is still generally believed that the exact computation of moments follows only from algebraic methods. Therefore, it is gratifying that the authors have shown an algebraic method which may be visualized and checked by graphical constructions, not only in the final result but also in successive steps of the computation.

ROBERT B. B. MOORMAN,[35] M. ASCE.—One of the disadvantages in the use of the method of analysis proposed in this paper is the evaluation of $A\bar{x}_A$ and $A\bar{x}_B$ in Eqs. 16 and 17. This may be overcome to a certain degree, for prismatic sections, for uniform loads extending over the span and for concentrated loads by use of the expressions:

$$A\bar{x}_A = \frac{w\,L^4}{24} + \tfrac{1}{6}\,\Sigma P(L - x)(L + x)x = \frac{w\,L^4}{24} + \tfrac{1}{6}\,\Sigma P(L^2 - x^2)x \quad (55a)$$

and

$$A\,\bar{x}_B = \frac{w\,L^4}{24} + \tfrac{1}{6}\,\Sigma P(L - x)(2\,L - x)x = \frac{w\,L^4}{24} + \tfrac{1}{6}\,\Sigma P(2\,L^2 - 3\,L\,x + x^2)x \quad (55b)$$

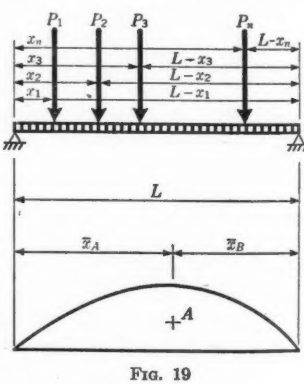


FIG. 19

The notation is defined in Fig. 19.

[33] *Transactions*, ASCE, Vol. 110, 1945, p. 1542.

[34] *Ibid.*, Vol. 111, 1946, p. 887.

[35] Prof., Civ. Eng., College of Eng., Univ. of Missouri, Columbia, Mo

It may be of interest to note that methods of analysis along somewhat the same line have been proposed by several others.[36,37,38,39,40]

EDUARDO AGRAMONTE,[41] ESQ.—The authors have achieved their purpose in presenting a method of direct analysis requiring the solution of few, if any, simultaneous equations; it requires computing stiffness and carry-over factors in order to distribute and to carry over moments. This is more than casual parallelism with other moment-distribution methods and, in fact, warrants the inclusion of the authors' method in the same group.

A single set of basic principles governs slope deflection, iterative and noniterative moment distribution, fixed points, balancing angles, traversing elastic curves, and similar methods. Procedures may vary in many ways, hence the diversity of methods. Some principles and their related operative factors play a prominent part in one group of methods whereas other principles and factors have a greater role in others.

Continuous frame analysis was in need of an expeditious method when Hardy Cross, M. ASCE, presented moment distribution.[8] Universal welcome and adoption are the best proof of its merits; since it is neither unorthodox nor inexact, there is no reason for considering it less classical, precise, or exact than others. Methods of the types presented by T. Y. Lin,[39] Assoc. M. ASCE, J. A. Wise,[42] M. ASCE, and the authors may adequately be grouped as direct, noniterative, or single-cycle moment distribution. Development in this field is in progress; closed frames were a stumbling block, as cyclic computation of stiffnesses, followed by cyclic distribution of moments, appeared inevitable. However, since rapid convergence of values to close degrees of precision was no comfort, the authors' "fixed joint" expedient is a valuable step forward.

Usual moment distribution is simple because prismatic members require only one K-value each, and carry-over factors are known to be 0.5 throughout. For symmetrical members with variable moment of inertia, one stiffness factor and one carry-over factor suffice. Unsymmetrical members require two of each; as a contrast, direct distribution starts with such factors and proceeds to determine two additional stiffnesses, and two more carry-over factors for every member. This amount is quite enough. Auxiliary factors, such as rotation ratios, moment ratios, the authors' "elastic length," and possibly the writer's J-factor (explained hereafter) defeat their purpose.

Direct distribution is especially convenient when dealing with variable moment of inertia. Iterative moment distribution becomes less convergent and its carry-over factors are no longer 0.5. In such cases, direct methods

[36] "Structural Frameworks," by Thomas F. Hickerson, Univ. of North Carolina Press, Chapel Hill, N. C., 1934.

[37] "Neue Statik," by Max Mayer, Bauwelt-Verlag, Berlin, 1937 and 1942.

[38] "Method of Moment Distribution by Means of Increased Moments," by Tsu Wu Chao, *Bulletin*, College of Eng., Kung Shang Univ., Tientsin, China, Vol. XI, No. 1, p. 13.

[39] "A Direct Method of Moment Distribution," by T. Y. Lin, *Transactions*, ASCE, Vol. 102, 1937, p. 561.

[40] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, John Wiley & Sons, Inc., New York, N. Y., 1932, p. 118.

[41] Civ. Engr., Ministry of Public Works, Buenos Aires, Argentina.

[8] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, ASCE, Vol. 96, 1932, p. 1.

[42] "A Precise Moment Distribution Method," by Joseph A. Wise, *Journal*, ACI, November, 1938, p. 89.

compare to best advantage. For these reasons, the writer feels justified in extending the authors' method to cover nonprismatic members.

Analysis of Individual Members.—If c_{AB} is assumed to be the ordinary carry-over factor from A to B (B fixed), Eq. 1 becomes, for the conditions shown in Fig. 2,

$$M_{BA} = c_{AB} \, M_{AB} \quad \text{(56)}$$

Also, if S_{AB} denotes the stiffness of end A when end B is fixed, the flexibility of end A (Eq. 3) will be:

$$Z_{AB(f)} = \frac{1}{S_{AB}} \quad \text{(57)}$$

Likewise, if U_{AB} denotes the stiffness of end A when end B is hinged, its value is:

$$U_{AB} = (1 - c_{AB} \, c_{BA}) \, S_{AB} \quad \text{(58)}$$

Inasmuch as

$$c_{AB} \, S_{AB} = c_{BA} \, S_{BA} \quad \text{(59a)}$$

$$c_{AB} \, U_{AB} = c_{BA} \, U_{BA} \quad \text{(59b)}$$

Eq. 5 becomes:

$$Z_{AB(h)} = \frac{1}{U_{AB}} \quad \text{(60)}$$

Fixed-end moments and c-factors and S-factors for various types of nonprismatic members are available in current literature. As hinged end stiffness factors, U, are decidedly superior to S-factors in direct distribution methods, their appearance in future diagrams or charts for nonprismatic members would be convenient.

The column analogy furnishes the same data for any type of loading and shape. Flexible connections may be treated as incidental to the elastic properties of the member to which they belong, by determining final fixed-end moments and c-factors, U-factors, or S-factors which take the connections into account.

Letting $Z^R{}_A$ and $Z^R{}_B$ be the flexibility of each connection of member AB and $c^o{}_A$, $c^o{}_B$, $U^o{}_A$, $U^o{}_B$, $\text{FEM}^o{}_A$, and $\text{FEM}^o{}_B$, be the factors and fixed-end moments for the member considered deprived of connections, final factors and fixed-end moments become:

$$U_A = \frac{U^o{}_A}{1 + U^o{}_A \, Z^R{}_A} \quad \text{(61a)}$$

$$c_A = \frac{c^o{}_A}{1 + U^o{}_B \, Z^R{}_B} \quad \text{(61b)}$$

and

$$\text{FEM}_A = \text{FEM}^o{}_A - \frac{U_A}{1 - c_A \, c_B} \left(Z^R{}_A \, \text{FEM}^o{}_A + c_A \, Z^R{}_B \, \text{FEM}^o{}_B \right) \quad \text{(61c)}$$

which are valid for end B by transposition of subindexes.

When one of both ends of a member should be considered infinitely stiff ($I = \infty$) and the central section is prismatic ($I = I_o$) the factors may be computed from Fig. 20, obtained by application of the column analogy.

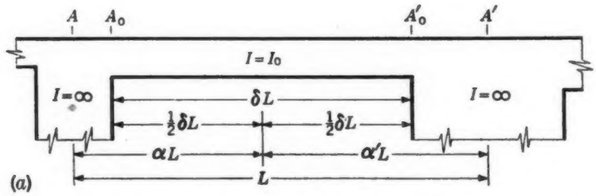
Frame Analysis.—To prevent confusion, the following sign convention is used throughout this discussion:

(*a*) Clockwise rotation is positive.

(*b*) End moments (couples) take the sign of rotation they produce or tend to produce; their sign is opposite to that of the rotations they resist.

(*c*) Bending moments along the span are positive where they produce tension at the under surface of beams or the right side of columns; therefore, when making moment diagrams the sign of all left-end beam moments or bottom-end column moments should be inverted.

(*d*) Shear is positive, where, should transverse shear rupture occur, particles imprisoned between the sliding fracture planes would be made to roll clockwise.



(*a*)

(*b*) STIFFNESS AND CARRY–OVER FACTORS

$3\,\alpha^2 + \left(\frac{\delta}{2}\right)^2 = m$	$m' = 3\,(\alpha')^2 + \left(\frac{\delta}{2}\right)^2$
$\frac{m}{\delta^3}\,\frac{4\,E\,I_o}{L} = S$	$S' = \frac{m'}{\delta^3}\,\frac{4\,E\,I_o}{L}$
$\frac{1}{\delta\,m'}\,\frac{3\,E\,I_o}{L} = U$	$U' = \frac{1}{\delta\,m}\,\frac{3\,E\,I_o}{L}$
$\frac{3\,\alpha}{m} - 1 = c$	$c' = \frac{3\,\alpha'}{m'} - 1$

$$\text{Check:}\quad \frac{S}{S'} = \frac{U}{U'} = \frac{c'}{c} = \frac{m}{m'}$$

(*c*) FIXED–END MOMENTS FOR UNIFORM LOAD, *w*, ON FULL SPAN

$(2\,m - \delta^2)\,\frac{w\,L^2}{12} = M_{fAA'}$	$M_{fA'A} = (2\,m' - \delta^2)\,\frac{w\,L^2}{12}$

FIG. 20.—CONSTANTS FOR MEMBERS HAVING INFINITELY STIFF END SECTIONS

In Fig. 21, a positive external moment M_{A-B} acts at end A of member AB causing its positive rotation, θ_A, which is resisted by end B with intensity $M_{AB} = -\,M_{A-B}$. Flexure, thus produced, causes end BA to exert a negative moment M_{BA} at joint B, which in turn induces negative rotation, θ_B. All other members (BC, BD, and BE) react in unison, opposing an aggregate negative moment $M_{B-A} = -\,M_{BA}$. The writer propounds subindexes such as "B–A"

as illustrative for moments or factors at joint B, pertaining to all members meeting at this joint except member BA.

If moment M_{A-B}, in Fig. 21, would originate rotation $\theta_A = +1$ radian, it would equal T_{AB}, the stiffness of end A of member AB. The reciprocal of T_{AB} is Z_{AB}; the rotation originated by unit moment $M_{A-B} = +1$—the flexibility of end A. If members AB, BD, and BE are suppressed and an external moment is applied at B to end BC, stiffness T_{BC}, or flexibility Z_{BC} of member BC, would be similarly illustrated.

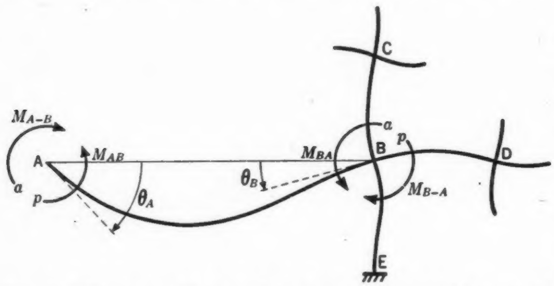


FIG. 21.—EXTERNAL MOMENT APPLIED AT END A OF MEMBER AB

Resistance to rotation at B, provoked by such a moment as M_{BA}, coming from member AB, depends on the aggregate stiffness of ends BC, BD, and BE, which is

$$T_{B-A} = T_{BC} + T_{BD} + T_{BE} = \frac{1}{Z_{B-A}} \ldots\ldots\ldots\ldots (62)$$

which in turn is equivalent to Eq. 13*b*. The total stiffness at B would be:

$$\begin{aligned} T_{B+} &= T_{BA} + T_{BC} + T_{BD} + T_{BE} = T_{B-A} + T_{BA} \\ &= T_{B-C} + T_{BC} = \cdots = T_{B-N} + T_{BN} \ldots\ldots\ldots\ldots (63) \end{aligned}$$

The authors provide no symbol for the carry-over or transmission factor.[43] Let such a factor be g; then $M_{BA} = g_{AB}\, M_{AB}$ in Fig. 21. Elastic lengths, flexibility, and carry-over factors for flexibly connected members are given in the following expressions—respectively, equivalent to Eqs. 24, 22, and 25:

$$L_{2AB} = (1 + Z^R{}_{AB}\, U^o{}_{AB})\, L \ldots\ldots\ldots\ldots (64a)$$

$$L_{3AB} = [1 + (Z^R{}_{BA} + Z_{B-A})\, U^o{}_{BA}]\, L \ldots\ldots\ldots\ldots (64b)$$

and

$$Z_{AB} = \frac{L_{2AB}\, L_{3AB} - c^o{}_{AB}\, c^o{}_{BA}\, L^2}{U^o{}_{AB}\, L_{3AB}\, L} \ldots\ldots\ldots\ldots (64c)$$

Also,

$$g_{AB} = c^o{}_{AB} \frac{L}{L_{3AB}} \ldots\ldots\ldots\ldots (65)$$

However, if values of c and U are previously computed to take into account the flexibility of connections, as in Eqs. 61, no further attention is necessary in that respect and Eqs. 66 to Eq. 86 apply directly.

Analysis of the frame may be accomplished by:

$$L_{AB} = (1 + U_{BA}\, Z_{B-A})\, L \ldots\ldots\ldots\ldots (66a)$$

and

$$Z_{AB} = \frac{L_{AB} - c_{AB}\, c_{BA}\, L}{U_{AB}\, L_{AB}} \ldots\ldots\ldots\ldots (66b)$$

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[43] "Moving Loads on Restrained Beams and Frames," by R. C. Brumfield, *Transactions*, ASCE Vol. 111, 1946, p. 807.

by following the operative sequence, Eqs. 13*b*, 66*a*, and 66*b*, obtaining *g*-factors independently from:

$$g_{AB} = c_{AB} \frac{L}{L_{AB}} \qquad (66c)$$

Or, by the operative sequence, Eqs. 62, 67*a*, and 67*b*

$$g_{AB} = \frac{c_{AB}\, T_{B-A}}{U_{BA} + T_{B-A}} \qquad (67a)$$

and

$$T_{AB} = \frac{U_{AB}}{1 - c_{BA}\, g_{AB}} \qquad (67b)$$

Or, since products $g\,T$ are as useful as single *g*-factors:

$$g_{AB}\, T_{AB} = \frac{c_{BA}\, U_{BA}\, T_{B-A}}{U_{BA} + (1 - c_{AB}\, c_{BA})\, T_{B-A}} \qquad (68a)$$

and

$$T_{AB} = U_{AB} + c_{BA}\,(g_{AB}\, T_{AB}) \qquad (68b)$$

may be found by the operative sequence, Eqs. 62, 68*a*, and 68*b*.

It may be found less tiresome, however, to avoid the repeated alternative use of expressions so dissimilar as types *a* and *b* of Eqs. 66, 67, or 68 by dealing, in the first place, with stiffnesses,

$$T_{AB} = \frac{(U_{BA} + T_{B-A}) - c_{AB}\, c_{BA}\, T_{B-A}}{(U_{BA} + T_{B-A})\, c_{AB}\, c_{BA}\, T_{B-A}} \qquad (69a)$$

by using Eqs. 62 and 69*a*, and to compute *g*-factors or ($g\ T$)-factors afterwards:

$$g_{AB} = \frac{T_{AB} - U_{AB}}{T_{AB}\, c_{BA}} \qquad (69b)$$

or

$$g_{AB}\, T_{AB} = \frac{T_{AB} - U_{AB}}{c_{BA}} \qquad (69c)$$

As a check on *T*-factors, *g*-factors, or ($g\ T$)-factors computed, obtaining for each member at the same time a new factor, *J*, that is useful for loading and sidesway analyses, these expressions hold true:

$$\frac{g_{AB}\, T_{AB}\, T_{A-B}}{T_{A+}} = J = \frac{g_{BA}\, T_{BA}\, T_{B-A}}{T_{B+}} \qquad (70)$$

If B is fixed, $T_{B-A} = T_{B+} = \infty$, and

$$\frac{g_{AB}\, T_{AB}\, T_{A-B}}{T_{A+}} = J = g_{BA}\, T_{BA} \qquad (71)$$

If B is hinged, $T_{B-A} = g_{AB} = J = 0$, but a check is offered by

$$\frac{c_{AB}\, T_{AB}\, T_{A-B}}{T_{A+}} = g_{BA}\, U_{BA} \qquad (72)$$

Factor *J* is also equivalent to

$$J = \frac{U_{AB}}{c_{BA}} \frac{g_{AB}\, g_{BA}}{1 - g_{AB}\, g_{BA}} = \frac{U_{AB}}{c_{BA}} \frac{L^2}{\dfrac{L_{AB}\, L_{BA}}{c_{AB}\, c_{BA}} - L^2} \qquad (73)$$

Transverse Loading.—Referring to Fig. 4, the moment diagram Fig. 4(*c*) considering the member as simply supported, must first be reduced, dividing its ordinates M by the local values of $E\,I$ along the span. Area A' and the location of the centroid (x_A, x_B) of the resulting diagram must be determined. Then, member AB, as actually connected to the frame, will exert moments M_{AB} at A and M_{BA} at B (opposite to those shown in Fig. 4(*b*) which amount to:

$$M_{AB} = A' \frac{U_{AB}}{c_{BA}} \frac{x_A\,L - \dfrac{x_B}{c_{AB}} L_{AB}}{\dfrac{L_{AB}\,L_{BA}}{c_{AB}\,c_{BA}} - L^2} \qquad (74)$$

which is equivalent to Eq. 16*a*. By inverting the subindexes M_{BA} is given. If the J-factor for the span is known,

$$M_{AB} = J \quad A' \left(\frac{x_A}{L} - \frac{x_B}{g_{AB}\,L} \right) \qquad (75a)$$

and

$$M_{BA} = J \quad A' \left(\frac{x_A}{g_{BA}\,L} - \frac{x_B}{L} \right) \qquad (75b)$$

When fixed-end moments are obtainable, no moment diagrams are necessary, since

$$M_{AB} = \frac{\text{FEM}_{AB}}{T_{A+}} T_{A-B} - \frac{\text{FEM}_{BA}}{T_{B+}} g_{BA}\,T_{BA} \qquad (76)$$

which also is valid for M_{BA} by transposition of the subindexes.

Sidesway.—Eqs. 20 take any of these alternative forms:

$$M_{AB} = \Delta U_{AB} \frac{L_{AB} + c_{AB}\,L}{L_{AB}\,L_{BA} - c_{AB}\,c_{BA}\,L^2} \qquad (77)$$

$$M_{AB} = \frac{\Delta}{L} \frac{1 + g_{AB}}{g_{AB}} J \qquad (78)$$

$$M_{AB} = \frac{\Delta}{L} \frac{(T_{AB} + g_{AB}\,T_{AB})\,T_{A-B}}{T_{A+}} \qquad (79)$$

and

$$M_{AB} = \frac{\Delta}{L} \left(\frac{T_{AB}}{g_{AB}\,T_{AB}} + 1 \right) J \qquad (80)$$

Transmission of Moments.—Stiffness T is, by definition, moment capable of provoking unit rotation; moments M_{A-B} and M_{BA} (Fig. 21) provoke rotations θ_A and θ_B, and the moment M_{CB} transmitted to C will provoke rotation θ_C. Hence,

$$\theta_A = \frac{M_{A-B}}{T_{AB}} \qquad (81a)$$

$$\theta_B = \frac{M_{BA}}{T_{B-A}} \qquad (81b)$$

and

$$\theta_C = \frac{M_{CB}}{T_{C-B}} \qquad (81c)$$

and such rotations awake the passive reactions:

$$M_{AB} = -\theta_A T_{AB} \qquad (82a)$$

$$M_{B-A} = -\theta_B T_{B-A} \qquad (82b)$$

$$M_{BC} = -\theta_B T_{BC} \qquad (82c)$$

and

$$M_{BD} = -\theta_B T_{BD} \qquad (82d)$$

and so on, for the other moments.

Distribution results from combining Eqs. 81 and Eqs. 82, which give

$$M_{B-A} = -M_{BA} \qquad (83a)$$

$$M_{BC} = -\frac{M_{BA}}{T_{B-A}} T_{BC} \qquad (83b)$$

and

$$M_{BD} = -\frac{M_{BA}}{T_{B-A}} T_{BD} \qquad (83c)$$

Carry-over (active) moments result either from distributed (passive) moments at the far end ($M_{CB} = g_{BC} M_{BC}$) or directly, in a single slide-rule movement, when distributing from active moments outside the far end, thus:

$$M_{CB} = -\frac{M_{BA}}{T_{B-A}} g_{BC} T_{BC} \qquad (84a)$$

and

$$M_{DB} = -\frac{M_{BA}}{T_{B-A}} g_{BD} T_{BD} \qquad (84b)$$

in which case products $g\,T$ apply throughout instead of single g-factors (see Eqs. 69, 70, 71, 76, 79, and 80). This also allows abbreviated distribution—by computing only active moments for the whole frame first, and passive moments when and where desired.

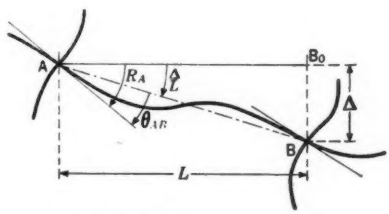

FIG. 22.—ROTATION AT JOINTS

Check on Final Moments.—The tangent at end A of a member AB (Fig. 22) forms an angle θ_{AB} with the straight line joining A and B such that:

$$\theta_{AB} = -\frac{(M_{AB} - \mathrm{FEM}_{AB}) - c_{BA}(M_{BA} - \mathrm{FEM}_{BA})}{U_{AB}} \qquad (85)$$

in which the fixed-end moments (FEM) are those due to all transverse loads on the member, and M_{AB} and M_{BA} are the actual moments at each end. This expression holds true with or without relative displacement of the joints. Rotation for any joint, computed separately by applying Eq. 85 to the end of each member framing into the joint, affords the desired check. In the case of displacements or sidesway, the quantity $\frac{\Delta}{L}$ must be noted as the check is in total rotation:

$$\left(\frac{\Delta}{L}\right)_{AB} + \theta_{AB} = R_A = \left(\frac{\Delta}{L}\right)_{AC} + \theta_{AC} \cdots = \left(\frac{\Delta}{L}\right)_{AN} + \theta_{AN} \ldots (86)$$

If actual rotations are desired in radians, proper values must be used for E and I; if not, arbitrary values do just as well.

In the practical solution of a problem, ordinary (iterative) distribution requires only one step less than that outlined herein; that is, frame analysis does not apply.

STEPHEN J. FRAENKEL,[44] ESQ., and ROBERT L. JANES,[45] JUN. ASCE.—An interesting new approach to the problem of calculating moments in rigid frames is presented in this paper. At the same time, the presentation of another method for solving stiff-jointed structures raises the basic question as to whether the proposed modifications of the commonly used analytical procedures offer such a significant improvement as to supplant present practices.

The advantage of this method as compared with the classical method of analysis is said to be the elimination of solving simultaneous equations in order to reach a solution. It is possible, however, that the difficulty of solving simultaneous equations has been somewhat exaggerated. In many cases the number of unknowns does not exceed five or six (see Example 3) and, in fact, two or three simultaneous equations will often suffice in a "least work" solution. Although the use of a slope deflection method in the case of a tall building becomes an absurdity, its application in a case of five or six unknowns does not seem unreasonable. It can be assumed, furthermore, that a computing machine is available in engineering offices where structural analyses are executed frequently, and a solution of six simultaneous equations on a machine is a small task. Even where longhand computations are required, the use of a method of successive approximation,[46] makes the solution of such a system of equations a relatively simple and quick matter.

A comparison between various methods of analysis, used in the Examples given by the authors seems an obvious method of judging relative applicability of each. To a small extent, such a comparison may be unfair to the authors, in that the examples chosen were for the purpose of illustrating the method, rather than for illustrating the type of problem most readily solved by it. Even a cursory check of Example 1, however, indicates that it could have been solved with much less effort by the three moment equation, moment distribution, or slope deflection. Example 2 could likewise have been done by the method of least work (requiring two simultaneous equations) or even more simply by a single slope deflection equation.

In view of present methods of analysis it is perhaps more pertinent to compare this method with those based on moment distribution, or with the method of moment distribution itself. It has not been claimed by the authors that this method affords a saving in time over the method of moment distribution, or, for that matter, over any other method, yet unless some advantage can be claimed there seems little practical benefit to be obtained. It is admittedly difficult to judge the relative merit, from a time standpoint, of various methods for accomplishing the same result, principally because the length of time required is intimately connected with the engineer's relative familiarity with the various methods. In spite of this inherent difficulty there appears to be no other satisfactory basis of comparison then a direct application to a

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[46] "Analysis of Rigid Frames," by A. Amirikian, U. S. Govt. Printing Office, Washington, D. C., 1942.

particular problem. A comparison has therefore been made using the authors' Example 4 as a basis. The methods used included the authors' method, that of moment distribution,[8] the modification or extension of this method as suggested by J. Wise,[47] and the column analogy.[48] The time required for a solution was considerably greater when using the method presented by the authors than by the use of any other methods. Granting a certain amount of lost motion due to unfamiliarity with the method, it seems doubtful, to say the least, that this time could have been equaled with the authors' method, even if one were thoroughly familiar with it. The discrepancy in time would probably be less in Examples 5, 6, or 7. In other words, it is believed that the procedure, as devised, sacrifices simplicity in simpler problems to applicability, and possibly superiority, when applied to complex frames. It must be remembered that the great majority of engineering calculations for indeterminate frames is relatively elementary and the cases in which the authors' procedure might excel constitute but a small minority of the total amount of work passing through a design office.

The suggested procedure would appear, therefore, to be a useful tool only to those who are constantly engaged in the analysis of complex stiff-jointed structures; the average structural engineer who deals with a wide variety of problems would in all likelihood not remember this procedure from one proper application to the next. This discussion is not meant to be a brief for mediocrity but a mere recognition of facts learned in structural design offices.

PHIL M. FERGUSON,[49] M. ASCE.—Although a different vocabulary and a somewhat different language are used in this paper, nevertheless, it seems to resolve itself into a restatement of the method introduced by T. Y. Lin,[39] Assoc. M. ASCE. The fundamental difference apparent between the two papers is the authors' use of an "elastic factor," Z. For the same purpose, Professor Lin uses a "modified stiffness," K_m. (He actually uses relative stiffness, the equivalent of substituting I/L for $4\,E\,I/L$.) If the unmodified K is assumed to represent $4\,E\,I/L$, the authors' Z is simply the reciprocal of K_m, the Lin modified stiffness factor. As a matter of fact, in the actual examples of this paper one finds Z always expressed as 1/2.500, 1/1.875, and so on. In other words, the authors find it convenient to tabulate the "stiffness factor" $1/Z$ instead of Z itself; and $1/Z$ is simply the modified stiffness, K_m, introduced by Professor Lin.

In like fashion, the authors' carry-over factors are identically those "modified carry-over factors" used in the Lin paper. It is true that this paper seems to use different formulas; but the formulas are different only because the authors begin with flexibility instead of its reciprocal, stiffness; also, they define and use a new quantity called "elastic length." Only a few simple algebraic steps are needed to show that the "different" formulas are, in reality, identities. The sequence of calculations for the frame constants also seems to be identical in the two methods.

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[47] "A Precise Moment Distribution Method," by J. A. Wise, Univ. of Minnesota, Minneapolis (mimeographed).

[48] "The Column Analogy," by Hardy Cross, *Bulletin No. 215*, Eng. Experiment Station, Univ. of Illinois, Urbana, 1930.

[49] Chairman, Civ. Eng., Univ. of Texas, Austin, Tex.

Likewise, in the handling of load terms, there is but one difference, which is a rather minor one. Professor Lin begins with fixed-end moments and must follow these with a single distribution and carry-over from each unbalanced joint before he obtains the authors' starting moments. The procedures seem to be identical from this point on.

A statement by the authors would be of interest, regarding the relative merits of these two approaches to this same general method—that is, the relative ease with which calculations or solutions can be formulated for more complicated problems. The writer is of the opinion that the basic moment-distribution process (essentially the same as that developed by Hardy Cross,[8] M. ASCE) will henceforth be the chief tool of designers in this field, and that all other methods will be secondary. In this situation he notes some advantages in the Lin approach to this problem. Modified stiffness and carry-over factors are closely related to the basic method in meaning and usage; and Professor Lin develops the factors very simply from it. This makes the development simpler for the student, and easier for the engineer who uses it only occasionally. Furthermore, the Lin formulas are adapted to members of variable section, inasmuch as all published tabulations of stiffness, carry-over factor, and fixed-end moment are directly usable. For variable sections, it would appear that the authors' method requires a return, each time, to the derivations based on the conjugate beam concept.

In the experience of the writer, very little use has been found for the term "flexibility," and it is about to be dropped from his teaching vocabulary. Since flexibility in algebraic terms is the inverse of stiffness, a new symbol and definition does not seem preferable to writing simply $1/K$ or $1/(4\,E\,K)$.

Part B of the authors' paper, with several examples of closed frames, is worthwhile in showing applications of this theory to frames more complicated than those previously covered by such methods. The device of a limited number of joints artifically restrained from rotation in the early stages is ingenious. A statement would be desirable as to whether the direct determination of the frame constants with all joints free to rotate was found to be a longer process, or whether it was ruled out as a method of successive approximation, and therefore not suitable for an "exact" solution. The tabulation forms are interesting and helpful; in fact, the authors are to be congratulated on a paper that is unusually clear and particularly well presented.

As a matter of general procedure, it appears that the multiple-panel Vierendeel truss of Example 6 could be more easily analyzed by the method of successive corrections. If one uses the criterion ratio procedure in such a solution (as was developed for wind stress calculations by L. E. Grinter,[50] M. ASCE, and if one is satisfied with errors as great as 1%, the time required should not be more than one third of that required for Example 6. Since this would not be an exact method, the comparison may not be fair.

[50] "Theory of Modern Steel Structures," by Linton E. Grinter, The Macmillan Co., New York, N. Y., 1936–1937, pp. 168–178.

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DISCUSSIONS

CLASSIFICATION AND IDENTIFICATION OF SOILS

Discussion

BY J. A. HAINE AND J. W. HILF, AND JACOB FELD

J. A. HAINE,[6] ESQ., AND J. W. HILF,[7] ESQ.—A development of soils classification for engineering purposes is presented to the profession by the author's excellent Airfield Classification (AC) System, which one of the writers studied under him at Harvard University in Cambridge, Mass., in 1943. From personal observation in the Southwest Pacific theater of operations, the AC system proved definitely valuable to United States Army engineer officers who were required to design airdromes rapidly in the field. The AC system was found to have two advantages over other current classification systems:

(1) The symbols used by Professor Casagrande are descriptive and easy to associate with actual soils, and experience in teaching this system to military personnel, who had no previous soils training, showed that it could be learned more quickly than systems depending solely on memory.

(2) The ability to assign definite engineering properties and even numerical values of design criteria, within reasonable limits, to the soil groups (such as the California Bearing Ratio) greatly facilitated the design of foundations for flexible pavements.

Although the AC system was devised for military roads and airdromes, it can be used profitably in foundation explorations and in prospecting for materials for rolled earth dams. In adapting this system to earth dams, it becomes necessary to expand two of the soil groups, and to specify certain limits of particle size and percentages that the author has not definitely stated in the paper.

Two of the fifteen soil groups, GF and SF, should be subdivided in order to distinguish the character of the fines in these soils which may be critical in high embankments because of the phenomenon of pore fluid pressures. This

NOTE.—This paper by Arthur Casagrande appeared in June, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1947, by Ralph E. Fadum; October, 1947, by James H. Stratton, and Donald J. Belcher.

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modification is easily accomplished without affecting the definitions of the original soil groups, by substituting GF (silty), GF (clayey), SF (silty), and SF (clayey), for GF and SF, respectively—mentioned by the author as a possible expansion of the AC system.

Specifications for rolled earth dams usually limit the maximum size of gravel in the rolled fill to 5 in. for a 6-in. compacted layer. In the investigation of sources of borrow materials for such structures, it follows that sizes larger than 5 in. should be excluded from the soil classification, although the importance of reporting oversize rock in exploration logs is recognized. With a range of coarse grains, from 0.1 mm to 5 in., the lower limit of gravel size (defined as 1 mm or 2 mm by the various grain-size classifications) should be raised to $\frac{1}{4}$ in. in order to simplify field identification, and to make the sand and gravel ranges more nearly equal on the grain-size curve. This definition more nearly corresponds with the coarse aggregate of concrete (No. 4 sieve), and was used by the Army (41).[7a]

A question often asked by those studying the AC system is, "How much gravel must be present in order to classify a coarse-grained soil as gravel, rather than as sand?" Professor Casagrande does not list quantitative limits for the soil groups in Table 4. Undoubtedly, such limits could be provided, but the percentages by weight would be different for each soil group, making it necessary to memorize additional information. Also, these percentages would be inappropriate in field classification, because of the difficulty of estimating weights of materials in various states of compactness by visual inspection. To avoid these objections and still provide a basis for dividing coarse-grained soils into sands and gravels, the writers have rearranged Table 4 for field use. As shown in Table 5, identification of coarse-grained soils (ten of the seventeen groups) may be accomplished by a step by step procedure without requiring the estimation of any fraction other than one half.

Col. 1, Table 4, lists fine-grained soils as "containing little or no coarse-grained material," but (under the heading, "12. Fine-Grained Soil Groups of AC System") the author states:

> "In the transition between coarse-grained and fine-grained soils, usually those soils containing more than 50% of material smaller than 0.1 mm in size (passing No. 150 mesh) are classified as fine-grained soils (ML or CL); and soils containing less than 50% smaller than 0.1 mm in size are classified as coarse grained (SF or SC)."

The writers agree with the latter division between the major soil groups and have used it in Table 5. The same reasoning is used in dividing coarse-grained soils into the gravel (G) and sand (S) groups. Regardless of the amount of fine-grained material present in coarse-grained soil, the latter is classified as gravel or sand, depending on the size of the coarse grains (whether more than 50% of them are larger or smaller than $\frac{1}{4}$ in.).

Persons who have not had the benefit of considerable laboratory experience in sieve analysis often find it difficult to estimate even the simple fraction,

[7a] Numerals in parentheses, thus: (41) refer to corresponding items in Bibliography (see Appendix of the paper), and at the end of discussion in this issue.

one half. They are inclined to judge quantity by volume, rather than by weight; for example, since the percentage by volume of particles larger than $\frac{1}{4}$ in. in a soil mass depends on whether the soil is in a loose state or a dense state, accuracy cannot be expected. However, the AC system boundary classifications, suggested by the author, such as GW-SW, or SF (silty)-ML, can be used in all cases where there is any doubt about the percentages.

Table 5 was devised to enable a relatively inexperienced investigator, or one unfamiliar with the AC system, to classify a soil—first, into the coarse-grained division or the fine-grained division, and then, if coarse-grained, into the gravel or sand groups when the soil is typical. For a boundary case in the coarse-grained division, the procedure is to assume that the soil is a gravel and then continue in the chart until the final soil group, say, GC, is reached, using the indicated criteria of gradation, and the amount and character of the fines. Since it could have been assumed that the doubtful coarse-grained soil was a sand, the correct field classification is GC-SC, which follows from the duplicate criteria for the gravel and sand subgroups. Similarly, if there is doubt as to whether a soil should be classified as coarse-grained or fine-grained, the assumption of first one type, and then the other, will result in the proper boundary classification. For fine-grained soils, Professor Casagrande's simple field tests, plus other identification aids, are used to determine the proper soil group. Here, the fraction to be tested is stated in Table 5 as material smaller than $\frac{1}{16}$ in., rather than No. 40 mesh. This is merely a field expedient for separation by hand, which will not affect the resulting classification in the large majority of cases, it is believed.

Together with instructions for performing the shaking, breaking, and thread tests, Table 5 should be sufficient to enable a proper classification in the field without any laboratory equipment. If it is desired, a representative sample can be classified in the laboratory, using the criteria in Table 5 and the Atterberg limits criteria proposed by Professor Casagrande; but it must be recognized that only a very small percentage of the soils examined in the field ever reach a laboratory, and that proper field classification is the primary objective in soils surveys. The writers heartily agree with the author that a complete description of the material is necessary in reporting a soil in a log. However, the facility in identification and classification of soils that must be developed by the use of such a chart as Table 5, is a necessary first step toward that goal.

After a soil is properly classified, as Professor Casagrande has shown, it is possible to indicate the engineering properties typical of the various soil groups and their use in engineering structures. For rolled earth dams, the permeability of a material is of outstanding importance and this property is listed for each group in Table 5. Also, the prospector for embankment materials often desires to know how the soil he has classified compares with other kinds of borrow material that may be available for a homogeneous dam, or for various zones of a zoned dam. To aid in evaluating possible sources of material, the writers have attempted to rate the various soil groups according to their experience, for use in rolled earth dams, considering the permeability, workability, shear strength, and resistance to piping of typical materials.

TABLE 5.—Soil Classification for Rolled Earth Dams (Adapted from Airfield Classification System)

1		2					3	4	5	6			
Major divisions		Field identification (Excluding material more than 5 in. in diameter)					Group symbol	Soil groups and typical names	Permeability	Relative Desirability for Embankment Use			
										Homogeneous	Zone 1[c]	Zone 2[c]	Zone 3[c]
Coarse-grained soils	Gravel and gravelly soils	More than 50% of sample is coarse grains (>0.1 mm[a])	More than 50% of coarse grains >¼ in.	Good gradation	Substantial amounts of all coarse sizes present (up to maximum size in sample)	Just enough clayey fines to bind coarse grains together	GC	Well-graded gravel-sand mixtures with excellent clay binder	Impervious	1	1	—	—
						"Clean" material (absence of appreciable amount of fines)	GW	Well-graded gravels and gravel-sand mixtures; little or no fines	Pervious	—	—	1	1
				Poor gradation	Predominantly more of one or more coarse sizes		GP	Poorly graded gravels and gravel-sand mixtures; little or no fines	Very pervious	—	—	—	2
					"Dirty" material (excess of fines)	Fines are silty (nonplastic)	GF[b] Silty	Very silty gravel, poorly graded gravel-sand-silt mixtures	Semipervious to impervious	2	2	2	—
						Fines are clayey (plastic)	GF[b] Clayey	Clayey gravel, poorly graded gravel-sand-clay mixtures	Impervious	3	3	—	—
	Sand and sandy soils		More than 50% of coarse grains <¼ in.	Good gradation	Substantial amounts of all coarse sizes present (up to maximum size in sample)	Just enough clayey fines to bind coarse grains together	SC	Well-graded sand with excellent clay binder	Impervious	4	4	—	—
						"Clean" material (absence of appreciable amount of fines)	SW	Well-graded sands and gravelly sands; little or no fines	Impervious	—	—	3	3[d]
				Poor gradation	Predominantly more of one or more coarse sizes		SP	Poorly graded sands and gravelly sands; little or no fines	Pervious	—	—	4	4[d]
					"Dirty" material (excess of fines)	Fines are silty (nonplastic)	SF[b] Silty	Very silty sands, poorly graded sand-silt mixtures	Semipervious to impervious	6	7	5	—
						Fines are clayey (plastic)	SF[b] Clayey	Clayey sands, poorly graded sand-clay mixtures	Impervious	5	5	—	—

TABLE 5.—(*Continued*)

1		2					3	4	5	6			
Major divisions		Field Identification (Excluding material more than 5 in. in diameter)					Group symbol	Soil groups and typical names	Permeability	Relative Desirability for Embankment Use			
		Visual	Tests on fraction < $\frac{1}{16}$ in. in diameter			Other means of identification							
			Shaking	Breaking	Thread					Homogeneous	Zone 1[c]	Zone 2[c]	Zone 3[c]
Fine-grained soils	Fine-grained soils of low to medium compressibility	More than 50% of sample is fine grains (<0.1 mm[a])	Moderate to fast reaction	Very slight to medium dry strength	Soft weak thread or none	Will form cracks when kneaded while moist	ML	Inorganic silts and very fine sands; rock flour; silty or clayey fine sands with slight plasticity	Semipervious to impervious	8	8	6	—
			Extremely slow reaction or none at all	Medium to high dry strength	Medium to tough thread	Can be kneaded while moist without cracking	CL	Inorganic clays of low to medium plasticity; sandy clays; silty clays; lean clays	Impervious	7	6	—	—
			Slow reaction	Slight to medium dry strength	Soft weak thread	Odor of decay which can be intensified by heating	OL	Organic silts and organic silt-clays of low plasticity	Semipervious to impervious	9	9	—	—
	Fine-grained soils of high compressibility		Slow to moderate reaction	Very slight to medium dry strength	Soft weak thread	Small shiny particles of mica or limestone can be seen	MH	Micaceous or diatomaceous fine sandy or silty soils; elastic silts	Semipervious to impervious	11	11	—	—
			No reaction	High dry strength	Very tough thread	In moist state, can be kneaded for a considerable period without cracking	CH	Inorganic clays of high plasticity; fat clays	Impervious	10	10	—	—
			No reaction	Medium to high dry strength	Soft weak to medium thread	Odor of decay which can be intensified by heating	OH	Organic clays of medium to high plasticity	Impervious	12	12	—	—
Fibrous organic soils of high compressibility		Fibrous, decayed, organic vegetable matter; noticeably compressible					PT	Peat and other highly organic swamp soils	Entirely unsuitable				

[a] 0.1 mm is the smallest visual size. [b] Subdivisions of the GF-groups and SF-groups of the AC system. [c] Zone 1 is inner, impervious zone; Zone 2 is intermediate, semipervious zone; Zone 3 is outer, free-draining zone. [d] If gravelly.

In Table 5, Zone 1 represents the impervious zone, Zone 2, the semipervious zone adjacent to Zone 1 in the dam, and Zone 3, the free-draining outer zone of a rolled earth dam. It is recognized that the numerical ratings are not rigid, but in a qualitative way they will enable the investigator to judge the suitability of a material for rolled earth dam construction.

For final design of a dam, a soil classification alone is entirely insufficient. Laboratory tests must be made on representative samples of the foundation, on the required excavation, and on the proposed borrow areas to determine, among other properties, the permeability, cohesion, angle of internal friction, and compressibility characteristics of the materials. Nevertheless, when adequate soil classifications and descriptions are available in the logs of exploration, the designer has at his disposal timely information which he can use in making preliminary estimates, in determining the extent of additional field investigations needed for final design, and in planning an economical laboratory testing program.

It would be desirable if there were a sufficiently large number of experienced soil engineers to personally perform all the identifications and classifications of soils which are required for earth dam projects, but the fact is that often the investigation is performed by personnel relatively inexperienced in soil mechanics. Under these conditions, the simplest possible descriptive classification system, consistent with adequate utility, appears to be desirable. The AC system without undue additional refinements is admirably suited for this purpose.

Jacob Feld,[8] M. ASCE.—This authoritative and complete program of soil classification is most timely. An extrapolation of reported experience to engineering problems is required to make the subject of soil mechanics usable in actual practice. To permit such procedure, the soil type must be known, when such experience is a report of someone else's work. A universal adoption of the author's system of classification and, at the same time, a recording of the soil type encountered in each problem reported in the technical literature will make a true wealth of data available in contrast to the present volume of contributions in soil mechanics; most contributions are useless because the authors employ secret codes or untranslatable language in describing the soils encountered and controlled.

The author clearly develops the thesis that soil classification must proceed on the basis of soil action and reaction, and not on the basis of component materials. The former method corresponds to the classification of human beings by their expected reactions, rather than by their height or weight. Textural or geometric soil classifications of granular or noncohesive soils have been reasonably successful, because the interaction of grains, in most cases, is not affected by the gas or liquid occlusions on the grain surfaces. In cohesive soils, there is a great variety of possible soil reactions, with identical textural make-up, depending on the mechanical and colloidal extent of the surface coatings. In addition, textural or sieving classifications do not distinguish between grains of unlike shape.

[8] Cons. Engr., New York, N. Y.

By magnifying the problem of soil classification, the difficulty of true classification by geometric methods can be easily understood. If a grouping of humans by size (let us say, equivalent volume) is made at a shore resort, and an identical size grouping is made at a winter resort, the two groups are geometrically identical; but they cannot be expected to react alike, because the factor of their clothing has been disregarded. Similarly, as the author described, the kaolin-type clays derived from a single type of feldspar may have particle sizes equivalent to many tough clays, but they react more nearly like a silt than a clay. The particles may be of the same size and, therefore, of equal gravitational adhesion; yet the chemical nature of the grain coverings and the effect on wetness (caused by the kind of electrostatic charge on the grain surfaces) are more important factors in determining the total adhesion and the tightness of possible grain packing, or the cohesion and possible consolidation, respectively.

In soil classification, it is unfortunate that the division between the sand and gravel designations does not agree with the accepted sieve separation in concrete and road base technology. The general agreement, that sand contains all particles passing a No. 4 sieve in all cement and bituminous concrete specifications, seems to require a restudy of this item for soil classification. To make the Airfield Classification (AC) System complete, definitions of the terms used, such as "compressibility" and "dry strength" should be included. It is important to distinguish between compressibility or dilatation as an elastic phenomenon, and changes in shape without change in volume, resulting from plastic flow or viscous deformation.

The author's recommendation of field identification tests brings to mind a very early treatise on soil classification, presented by John Evelyn before the Royal Society of London almost three hundred years ago (42). Although he was chiefly interested in the agricultural possibilities in artificially prepared soils (as well as natural soils), the recommendations for classifications and field identification are quite similar to recent writings, and are summarized herein for the reader who does not have access to this rare book.

In the "Introduction," Evelyn cites "De Arte Combinatoria" by Athanasius Kircher in which it is stated that there are "* * * no fewer than 179,100,060 different sorts of earths, but only eight or nine are useful to our purpose." Corresponding to modern soil horizons, three strata are identified as: (1) The top layer of "underturf earth," a foot or so deep; (2) the next strata may be "loam, clay, plastic, figuline or smectic"; and (3) the lowest strata may be "chalk, marle, fullers-earth, sandy, gravelly, stony, rock, shelly, coal or mineral." Incidentally, Evelyn states, "The Ancients called them: Creta, Argilla, Smectica, Tophacea, Pulla, Alba, Rufà, Columbina, Macra, Cariosa, Rubrica." He further subdivides the top layer into:

"I. A virgin-earth, black, fat, porous, light and sufficiently tenacious, without any admixture of sand or gravel, which in the lower series may be darkest-gray or tawney, becoming veined with yellow or sometimes red.
II. An obscure color mould, with some loam and sand.
III. A mixture of the former two types with small flints and pebbles.
IV. A totally sandy with a bottom of gravel, rock and 'not seldom' clay."

Then, a general description of soil types follows, which is quoted in full because of its picturesque language:

> "Pure sand is white, black, bluish, red, yellow, harsher and milder and some mere dust in appearance, none of them to be desired alone. But the grey black and ash coloured, or of the travelling kind, volatile and exceedingly light, is the most insipid and worst of all.
>
> "Clays there are of many Kinds—some more pinguid, some more slippery, all of them tenacious of water on the surface where it stagnates without penetration, most of them pernicious and untractable. The blue, white and red clay are all unkind, the stony and looser sort is sometimes tolerable, but the light brick-earth does very well with most fruit trees.
>
> "Loams and brick earth are of several sorts, some approaching to clay, others nearer marle.
>
> "Marle is usually at greater depths, of many colours, all of them unctuous, slippery nature, slackens upon drying after a shower and crumbles into dust."

Evelyn than advised that general textural description of soils may be of less value than classification of properties by the use of the senses. Four tests are described:

(1) By odor or smell—since "Lord Verulam affirms that vegetable odours exist in the soil."

(2) By taste—since the water percolation through soil shows the kind of chemicals therein, "although some say that there is no taste or odour in the best earths."

(3) By touch—fatty and slippery, or gritty, porous and friable, if it stick to the fingers or dissolve on the tongue. The "best earth is blackish, cuts like butter, sticks not obstinately, is sweet, and becoming mortar when wet, without crusting when dry."

(4) By sight—"grey is pre-eminent, next russett, clear tawney is worse, light and dark coloured are good for nothing, yellowish red is worst of all."

Having analyzed and shown how various basic properties could be identified, Evelyn did not entirely discredit the laboratory technician of his day; he advised the following procedure for the synthesis of desired properties in soils:

> "All these are fit to be known, as contributing to noble and useful experiments, upon due and accurate comparisons, and inquiries from the several particles of their Constitutions, Figures and Modes, as far at least, as we can discover them by the best auxiliaries of microscopes, lotions, strainers, calcinations, triturations and grindings; upon such discovery to judge of their qualities, and by essaying variety of mixtures, and imitating all sorts of mold, foreign or indigen, to compound earths as near as may be resembling the natural, for any special or curious use, and be thereby enabled to alter the genius of Grounds as we see occasion."

He then demonstrated how artificial soils had been made by mixing samples of earths from various sources with types of chemicals and fertilizers, and concluded:

"Therefore let no man be over-confident, that because some earths are soft, fat and slippery, they may not possibly consist of sands, (of which there are so many Kinds), since 'tis evident, that even all fossile bodies which can be reduced and brought to sands, may by contrition of the particles be rendered so minute, as to emulate the finest earths we have enumerated."

The early French experimenters and theorists in soil problems distinguished only between "loose soils" and "firm soils." However, when M. Chauvelot (43) presented a paper before the French Academy of Sciences in 1783, showing how the lateral earth pressure could be derived from a knowledge of the weight, the internal friction, and the cohesion of a soil without any other description or classification, that learned body announced that the problem of the determination of lateral earth pressure was now susceptible to a rigorous solution. Later, contributors went back to a textural soil classification and M. Mayniel (44), in 1808, reports on his extensive experiments and tables to cover all kinds of soil, namely: Soil, sand, gravel, rubbish or ash, clay and debris fills. Incidentally, Mayñiel's division between large and small gravel is denoted by the size of a pigeon's egg.

An anonymous author (45) quotes from the works of Sir Humphrey Davy, classifying the four earths found in soils as: "* * * aluminous (clay), siliceous (flint), calcareous (limestone) and magnesian (not to be mistaken for limestone)." However, a soil is to be classified on how it acts and not on what it contains.

To quote that author:

"In framing a system of definitions, a soil is to take a particular designation from a particular Kind of earth, not exactly in proportion as that earth may preponderate, or not over others in forming the basis of the soil, but rather in proportion to the influence which a particular Kind of earth, forming part of the sample, has on tillage and vegetation."

That might also be said for the influence on engineering uses of soils.

The need for a universal soil classification is an old one and much credit must be given to the author of this paper in presenting a comprehensive answer to it, in the form of the AC system, which permits and provides a unique name for every type of soil, using expected action and reaction as criteria.

Bibliography.—

(41) **"Aviation Engineers,"** U. S. War Dept., *Technical Manual TM 5-255*, U. S. Govt. Printing Office, Washington, D. C., April 15, 1944.

(42) **"Terra, a Philosophical Essay of Earth,"** by John Evelyn, London, 1678.

(43) **"Traite Experimental et Analytique de la Poussee des Terres,"** by M. Mayniel, D. Colas, Paris, 1808, p. 102.

(44) *Ibid.*, Book IV, p. 166.

(45) **"Treatise on Soils and Manures,"** by a Practical Agriculturist, London, 1818.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

TESTS OF TIMBER STRUCTURES FROM GOLDEN GATE INTERNATIONAL EXPOSITION

Discussion

BY MELVIN W. JACKSON, AND CHARLES MACKINTOSH

MELVIN W. JACKSON,[13] JUN. ASCE.—The San Francisco Timber Test Program Committee calls attention to the necessity of considering tension perpendicular to grain for the correct design of timber joints, but it does not suggest how this may be taken into consideration. The problem raised is apparently an unsolved one. In testing approximately 250 ring-connectored timber joints of the type shown in Fig. 86, the writer obtained some information pertinent to this case.[14] The test specimens were intended to simulate conditions in a truss joint of the type shown in Fig. 36. The investigation was partly an outgrowth of a previously published report of this type of failure.[15]

In the test specimens (Fig. 86) member Nos. 1, 3, and 5 were end-matched 2 in. by 8 in. by $18\frac{1}{2}$ in. long, and the cross members, A and B, were similarly end-matched, but varied in dimensions. The lower bolt was provided to prevent excessive spread of member Nos. 1 and 5 under load. Standard procedure was followed in the laboratory in installing 4-in. split-ring connectors in the faces between each member. The test specimens, to which the data given hereafter apply, were of Douglas fir, structural grade (allowable working stress = 1,200 lb per sq in.). Two general types of failures occurred in members A and B; that is, either cores sheared, or the horizontal members split longitudinally. The tests were stopped at the first indication of failure to determine the type of initial failure. It was necessary to take the joint apart to determine whether a core had sheared.

NOTE.—This paper by a Committee of the San Francisco (Calif.) Section, ASCE, on Timber Test Program was published in May, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1947, by James H. Carr, Jr., Howard J. Hansen, and E. G. Stern.

[13] Champaign, Ill.

[14] "An Experimental Investigation of Some Properties of Five-Member Ring-Connectored Timber Joints," by Melvin W. Jackson, thesis presented to the Univ. of Illinois at Urbana, in 1945, in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering.

[15] "Preventing Failure of Timber Joints," by Charles Mackintosh, *Civil Engineering*, December, 1943, p. 573.

Tension-perpendicular-to-grain tests following the recommended procedure of the American Society for Testing Materials were made from members A and B, and these values and the load at initial failure in the five-member specimens were compared. Where the horizontal members were 2 in. thick (nominal dimensions) and 20 in. or more long, and where the initial failure occurred by splitting, the correlation shown in Fig. 87 was obtained. Fig. 87 indicates that there is some correlation between the strength of a timber in tension perpendicular to grain, and the load which will cause it to split in a joint of the type tested. It is apparent, however, that the problem in this case cannot be solved simply by dividing the total stress in tension across the member by an allowable working stress in tension perpendicular to grain, because doubling the strength of the material in tension perpendicular to grain does not double the load required to produce initial failure. Possible explanations of this fact may be that the law of stress distribution within the member is not known or that the member and the control tests for establishing the tension-perpendicular-to-grain values are subject to stress concentrations not evaluated. On the other hand,

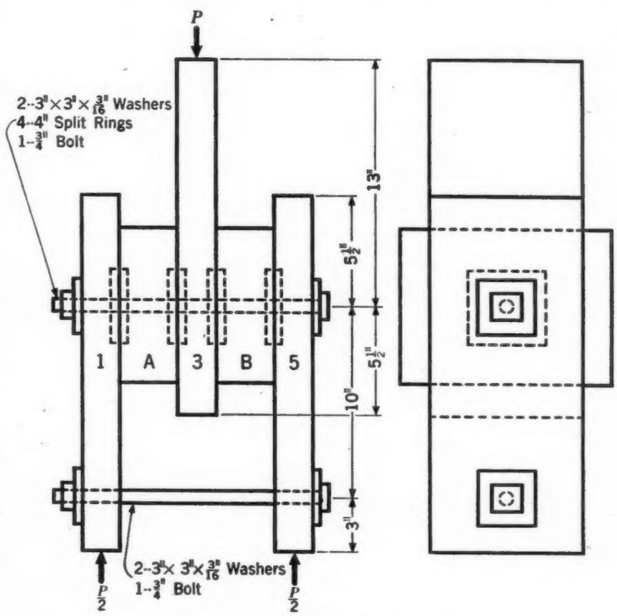

Fig. 86.—Assembly of Test Joint

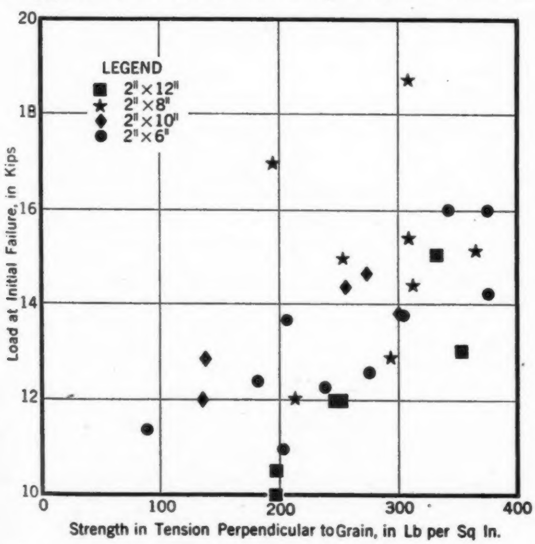

Fig. 87.—Comparison of Load at Initial Failure in 5-Member Test Joint with Tests of Tension Perpendicular to Grain in 20-in. and Longer Members

there may be other factors, such as those outlined in the notes in Fig. 36, which are not usually considered in joint design.

The committee presents a good summary of forces acting on the typical joint in Fig. 36. It should be kept in mind that the forces shown are under very low loads. Working loads or higher loads add to the complexity of the problem. At such loads there would also be bearing stresses under the bolt, and a bending moment applied to the bolt with resisting stresses under the washers. In addition, the horizontal member would rotate, the magnitude of this rotation depending on the amount of crushing of fibers under the forces (F_c) acting as shown, and under the washers in turn, depending on the fit of the members in the joint. This rotation further causes the rings to be displaced from their original position in the grooves and adds additional lateral bearing stresses under part of the ring edges.

CHARLES MACKINTOSH,[16] ESQ.—Although timber is one of the oldest structural materials used by man, the engineer's knowledge of its proper use in large frame structures is appallingly meager. This paper makes a signal contribution to the understanding of the actions of timber joints. It makes unmistakably clear the fact that the strength of the bolt, and split-ring (or other) connection in a given joint is dependent not upon the tables of values of such connecting devices, but rather upon the nature of the connection itself and the timber in the connection.

The tables of safe loads for bolts, split rings, and other connections are, in general, based upon the expected safe load when the failure is at the maximum value obtained from the most advantageous position of the connector. These tables do not generally anticipate the failure of the timbers in the connection. It is stated in the "Foreword," that nearly one half of the failures occurred "* * * because of types or combination of stresses not anticipated"—which is a most revealing evaluation of the present quality of engineering design.

The importance of secondary stress due to bending in the shallow trusses of the Brazil Building is well emphasized. This writer has found that similar importance should be attached to any condition where concentrated long-time loads occur on the chords between panel points of normally proportioned trusses. A heavily loaded beam supported between panel points on the lower chord of a timber truss may cause such a deflection over a long period of time that enough additional stress is caused by eccentricity to produce failure.

In discussing the failure of the cantilever frame in the Cavalcade Building, the paper emphasizes that oversized bolt holes may have contributed to the failure of the joint by causing some successive loading. If the effect of the oversized hole is so great when shear plate connectors are used, should not the manufacturer's recommended value be questioned? These values are based upon ultimate loads having greater movement than the tolerance caused by enlarged bolt holes. It is clear that final failure will occur if the loads which caused an initial movement of $\frac{1}{8}$-in. are maintained for a sufficient length of time. Such failure is attributed by some timber research men to "creep" or

[16] Cons. Engr., Mackintosh and Mackintosh, Los Angeles, Calif.

"progressive failure."[17] In the data published relative to split rings, "* * * the loads listed as maximum are the highest loads obtained within or at a slip in the joint of 0.60 inch, beyond which the tests were continued."[18] Is not 0.60 in. much too great a slip to give any worthwhile data for the design of connector joints subject to long continued load?

In discussing the heel failures of the trusses in the Alameda-Contra Costa Counties Building, the paper states (under the heading, "Detailed Report of Tests,") that: "Failures in the blocks at these relatively low unit [shearing] stresses were probably due to the abrupt change in sections * * *." Reentrant grooves in timber for various connectors cause abrupt changes in section and greatly influence the relative safe values of long-time and short-time loads. Unfortunately, few data are available on any type of connector stressed by long-time loads. Such tests have been initiated in both Australia and the United States, but are of such recent origin that final reports are not available. Oversized holes have long been prohibited by skilled timber truss manufacturers because of the large deflection in trusses with oversized holes, rather than because of the reduced value of the connection.

The necessity of considering tension perpendicular to grain and proper end distance and spacing has been well presented by the committee, and these comments point to more complete analysis than most engineers have been accustomed to making. This analysis must be supplemented by additional series of tests such as those sponsored by most universities.[14] The fullest use of timber cannot be made until such test data are available. A practical solution may lie in a type of connector which obtains more uniform attachment to the to the wood and does not require large end and edge distances.

Because of timber truss failures in Los Angeles, Calif., and particularly the failure of a splice in a truss using shear plates, the Superintendent of Building and Safety of the City of Los Angeles requested recommendations from the Structural Engineers' Association of Southern California. The subsequent report of this association and its Committee on Timber Research puts into usable form some of the conclusions to which the paper points.

The report recommends wider use of grade marking, straight grain tension members, the rejection of previously recommended methods[19] for computing spaced column values, greater attention to the moisture contents of lumber, calculation of tension-across-grain stresses and stresses due to eccentricity, and a reduction of connector values below that recommended by the manufacturers.

The authors of this paper are to be commended for the material aid that they have given to the development of a better understanding of structural design in wood.

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[17] "Green Timbers in Australian Structures," by Ian Langlands, *Engineering-News Record*, September 6, 1945, p. 94.

[18] "Timber-Connector Joints: Their Strength and Design," by John A. Scholten, *Technical Bulletin No. 865*, Forest Products Laboratory, Forest Service, U. S. D. A., Madison, Wis., March, 1944, p. 39.

[14] "An Experimental Investigation of Some Properties of Five-Member Ring-Connectored Timber Joints," by Melvin W. Jackson, thesis presented to the Univ. of Illinois at Urbana, in 1945, in partial fulfilment of the requirements for the degree of Master of Science in Civil Engineering.

[19] "Wood Structural Design Data," Supplement No. 4, National Lumber Manufacturers' Association, Washington, D. C., 2nd Ed., 1939.

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DISCUSSIONS

STABILITY OF THIN CYLINDRICAL SHELLS IN TORSION

Discussion

BY GLENN MURPHY

GLENN MURPHY,[28] M. ASCE.—This is an interesting approach to a timely problem, in that the author's analysis leads to results which not only are in reasonable agreement with representative tests, but which also tend to substantiate the theories of previous investigators. A number of conclusions may be drawn from Fig. 5, which gives values of K_D for use in Eq. 25*b*. It is apparent from Fig. 5 that there are two separate phases of behavior—one, in which $\left(\text{with constant } \frac{D}{t}\right)$ K_D, and consequently τ_c, decreases with increasing values of $\frac{L}{D}$; and the other, in which K_D and τ_c are independent of $\frac{L}{D}$. These two phases, between which there is shown smooth transition, confirm previous conclusions that one expression may be used for short cylindrical shells and a different expression for long shells.

In the range in which values of $\frac{L}{D}$ are large and the curves of Fig. 5 are horizontal, the equation for K_D is readily determined as:

$$K_D = 0.70 \sqrt{\frac{D}{t}} \qquad (37)$$

If this value of K_D is substituted in Eq. 25*b*, it becomes:

$$\frac{\tau_c}{E} = 0.70 \left(\frac{t}{D}\right)^{\frac{3}{2}} \qquad (38a)$$

which is exactly equivalent to the equation given by S. Timoshenko for long cylindrical shells with Poisson's ratio of 0.33. L. H. Donnell's equation reduces to

$$\frac{\tau_c}{E} = 0.80 \left(\frac{t}{D}\right)^{\frac{3}{2}} \qquad (38b)$$

NOTE.—This paper by R. G. Sturm was published in April, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: October, 1947, by S. B. Batdorf and Manuel Stein.

[28] Prof. of Theoretical and Applied Mechanics, Iowa State College, Ames, Iowa.

In the short-tube range an approximate equation may be developed for K_D by replacing the author's curves (formed from a series of loops) by a family of straight lines tangent to the curves. Whereas the slopes of these straight lines vary somewhat, none deviates appreciably from 0.55, and the equation of the family of lines may be determined as:

$$K_D = \left(\frac{D}{L}\right)^{0.55} \left(\frac{D}{t}\right)^{0.80} \qquad (39)$$

If this value of K_D is substituted in Eq. 25*b*, the latter becomes:

$$\frac{\tau_c}{E} = \left(\frac{D}{L}\right)^{0.55} \left(\frac{t}{D}\right)^{1.20} \qquad (40)$$

The Donnell equation for short tubes in torsion may be reduced to the form:

$$\frac{\tau_c}{E} = 0.75 \left(\frac{D}{L}\right)^{0.50} \left(\frac{t}{D}\right)^{1.25} \qquad (41)$$

It is apparent that Eqs. 40 and 41 are similar although not identical.

The approximate boundary between the short-tube range and the long-tube range may be established from Fig. 5 by noting that the locus of the points of intersection of the family of lines for the short cylinders with the corresponding $\left(\text{same } \frac{D}{t}\right)$ lines for the long cylinders is a straight line sloping upward to the right. The equation of this line may be determined as:

$$\frac{L}{D} = 2.45 \sqrt{\frac{D}{t}} \qquad (42)$$

If $\frac{L}{D}$ is less than $2.45 \sqrt{\frac{D}{t}}$, the shell may be considered short and Eq. 40 applies; whereas if $\frac{L}{D}$ is greater than $2.45 \sqrt{\frac{D}{t}}$, the shell may be considered long and Eq. 38*a* applies.

An interesting comparison may be made between Eq. 42 and the Donnell equation defining the boundary between the short tube and the long tube. For simply supported edges, Mr. Donnell establishes the boundary by

$$\frac{1}{\sqrt{1 - \mu^2}} \frac{L^2 t}{D^3} = 5.5 \qquad (43)$$

or

$$\frac{L^2 t}{D^3} = 5.19 \qquad (44)$$

for Poisson's ratio equal to 0.33. However, Eq. 42 may be in the form:

$$\frac{L^2 t}{D^3} = 2.45^2 = 6 \qquad (45)$$

which compares favorably with Eq. 44. It should be noted that Eqs. 40 and 42, developed by the writer from Fig. 5, are approximate and are presented primarily to indicate qualitative agreement between the results of the author and other investigators.

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DISCUSSIONS

FRICTION COEFFICIENTS IN A LARGE TUNNEL

Discussion

BY J. N. BRADLEY AND S. P. WING

J. N. BRADLEY,[42] ASSOC. M. ASCE, AND S. P. WING,[43] M. ASCE.—The authors and others concerned are to be commended on the meticulous manner in which the planning and the testing were performed on the hydraulic features of the Apalachia Tunnel. In making field tests the usual procedure is to assign to the job a man who is not especially trained in hydraulic testing, with instructions to acquire help in the vicinity and to perform a definite series of tests in the shortest possible time. As a result, instrument taps are placed in the most convenient points along the pipe rather than in the more significant locations, and difficulties are often encountered with the work performed by the untrained personnel. Information resulting from the tests is too often questionable and is at times contradictory. Tests on prototype structures are a bountiful investment when performed with care and a thorough understanding of the problem involved.

In the case of the Apalachia Tunnel tests, pressure taps were installed in significant locations along the tunnel at the time of construction and well-trained personnel performed the test work and analyzed the data. The results should be very dependable.

The writers and others of the Bureau of Reclamation have collected and compiled test data on friction coefficients for large pipe lines for a number of years. It is felt that some of this information should be of value to the engineering profession as a whole. For example, data showing the relation of the friction coefficient to Reynolds' number are presented in Fig. 19. The data contained in Fig. 19 represent results from concrete pipe to 18 ft in diameter, velocities as high as 42 ft per sec, and Reynolds' numbers approaching 30,000,000.

As each curve in Fig. 19 is numbered, by referring to Table 14, information can be found concerning the diameter, velocity, length, age of pipe, character

NOTE.—This paper by G. H. Hickox, A. J. Peterka, and R. A. Elder was published in April, 1947, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1947, by Weston Gavett; September, 1947, by W. R. Barrows, Hunter Rouse, Karl R. Kennison, E. J. K. Chapman, Julian Hinds, and William P. Creager and Stephen H. Haybrook; and October, 1947, by G. S. Tapley.

[42] Hydr. Engr., Bureau of Reclamation, Denver, Colo.

[43] Civ. Engr., Bureau of Reclamation, Denver, Colo.

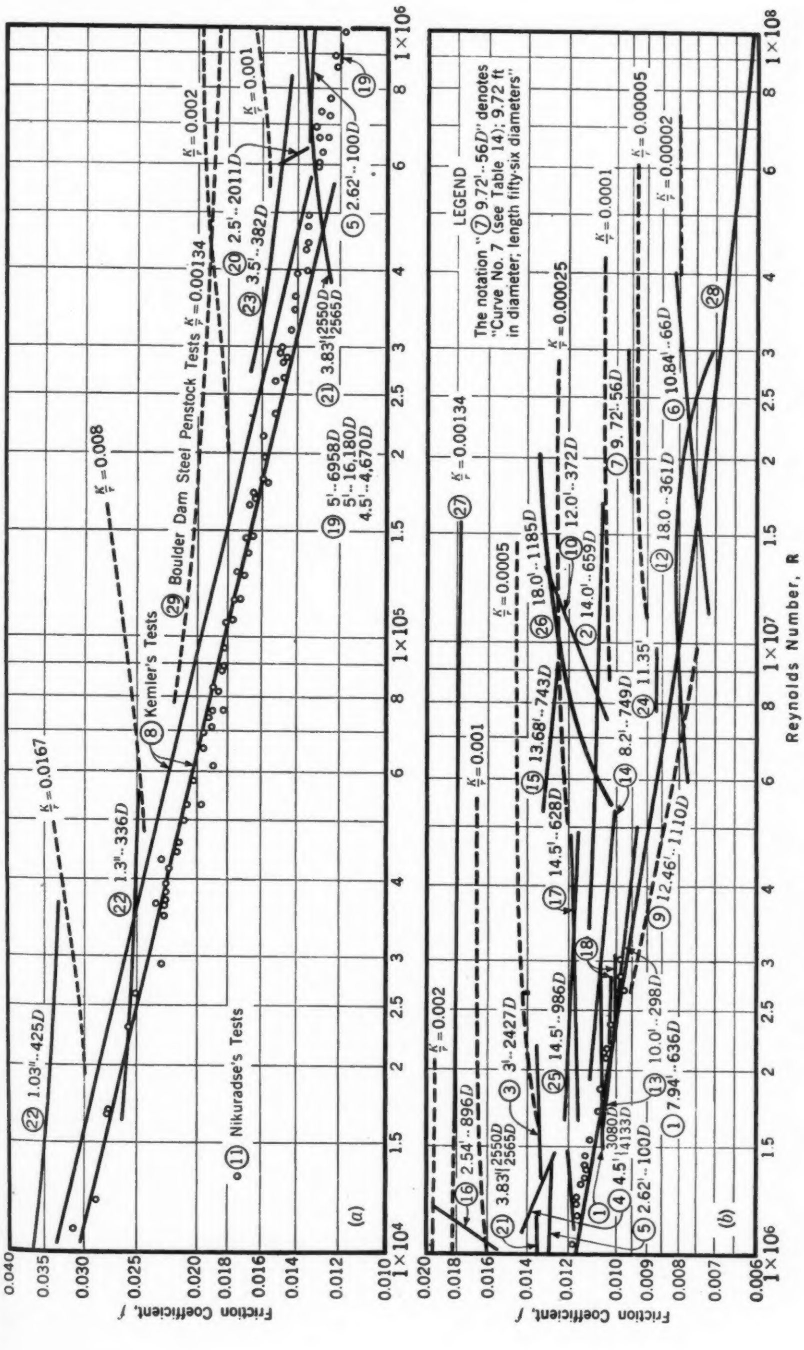

FIG. 19.—FRICTION FACTOR f VERSUS REYNOLDS' NUMBER **R** FOR LARGE CONCRETE PIPE

TABLE 14.—COMPARISON OF TEST

Curve (see Fig. 19)	Name and location	Age at test (yr)	Diameter (ft)	Velocity (ft per sec)	Length: Ft	Length: Diameters
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	Power plant, Castelleto, Italy	10	7.94	2.61 to 4.19	5,062	636
2	Chelan station, State of Washington	New	14.0	1.25 to 14.8	9,227	659
3	Deer Flat, Boise Project, Idaho	6	3.0	5.45 to 9.06	7,282	2,427
4	Water conduit No. 10, Denver, Colo.	4 1	4.5 4.5	2.58 to 3.50 2.58 to 3.50	18,598 13,902	4,130 3,089
5	Dijon, France	New	2.625	3.01 to 6.59	262.5	100
6	Englewood Dam, Ohio	1 to 4	10.84	17.6 to 42.2	712	66
7	Germantown Dam, Ohio	1 to 4	9.72	20.2 to 36.5	546	56
8	Reported by E. Kemler, Pennsylvania	....	0.009 to 0.438	....	....	1,500 to 2,000
9	Power plant, Livenza, Italy	2	12.46	2.76 to 10.17	13,850	1,111
10	Power plant, Melones, Calif.	....	12	7.60 to 13.49	4,469	372
11	Reported by J. Nikuradse, Germany	....	0.03 to 0.238	....	....	....
12	Tunnel, Ontario Power Company, Niagara Falls, Canada	8	18.0	4.0 to 2.00	±6,500	361
13	Power plant, Partidor, Italy	18	10	2.2 to 6.8	2,985	298
14	Power plant, Piavi-Ansiel, Italy	1	8.2	3.28 to 9.31	6,150	769
15	Power plant, Pit Dam No. 1, California	New	13.68	4.7 to 8.2	10,160	743
16	Prosser pressure pipe, Yakima, Wash.	4	2.54	4.9 to 5.8	2,276	896
17	Rondout siphon, Catskill Aqueduct, New York	New	14.5	1.6 to 4.8	9,102	628
18	Reported by E. W. Spies, Germany	....	....	....	....	....
19	Water pipe line, Spavinaw Aqueduct, Tulsa, Okla.	New	5 5 4.5	2.25 2.25 2.63	34,788 60,998 21,047	6,958 16,180 4,670
20	Umatilla Dam siphon, Umatilla Project, Oregon	New	2.5	3.4 to 3.6	5,026	2,011
21	Umatilla River siphon, Umatilla Project, Oregon	5 2	3.83 3.83	1.4 to 3.2 4.0 to 4.2	9,774 9,831	2,550 2,565
22	Hose-formed conduit, Colorado	New	0.086 0.108	1.9 to 7.2 2.1 to 7.2	36.5	424 336
23	Aqueduct, Victoria, B. C., Canada	2	3.5	1.0 to 2.9	1,336	382
24	Waggitaler, Germany	....	11.35	....	....	....
25	Wallkill siphon, Catskill Aqueduct, New York	New	14.5	1.6 to 4.8	14,300	986
26	Apalachia Tunnel, TVA, Tennessee	New	18	4.2 to 12.6	21,380	1,185
27	Experimental data	....	....	....	....	....
28	von Kármán-Nikuradse data	....	....	....	....	....
29	Steel penstock, Boulder Dam, Colorado	....	....	....	....	....

Description and literary reference

(8)

Curve 1. Smooth cement surface; discharge rated by a current meter placed in the tailrace; about 10% of the line located on a small degree of curvature ($r/D = 30$). About 5% subtracted from the over-all measured losses as an estimate of entrance losses. The writers estimate that the plotted losses are about 1% greater than normal. This was a reliable test. ("Correnti Uniformi entro grandi condotte e grandi canali, Milano," by Giuliode Marchi, Milan, Italy, 1932–1936; Library Data File, U.S.B.R., 91–241.)

Curve 2. Smooth surface resulting from use of steel forms; discharge rated by Gibson method. The section measured was straight. The velocities are probably 3% in error. Combining all errors, the friction factor was probably not more than 4% in error. (*Transactions*, ASCE, Vol. 101, 1936, p. 1409; also Library Data File, U.S.B.R., 91–241.)

Curve 3. Pre-cast in steel forms 6 ft long; discharge rated by color movement, current meter, and Cipoletti weir. All joints were carefully calked on the inside. The alinement was straight. There was a gentle vertical bend near the inlet and one near the outlet (*Bulletin No. 852*, by Fred C. Scobey, U.S.D.A., Washington, D. C., 1924, p. 38.)

Curve 4. Lining and the joints smooth; discharge rated by pitot tube. The alinement of the first section was nearly straight and there was a gentle sinuous curve, vertically, in the second section. This pipe was pre-cast in 12-ft oiled steel forms. (*Engineering News-Record*, April 29, 1926, p. 678.)

Curve 5. These experiments were reported by Bazin to be "perfect in bore." The alinement was straight and the results indicate an unusually smooth pipe. (*Bulletin No. 852*, U.S.D.A., Washington, D.C., 1924, p. 79.)

Curve 6. Lining finished with a brush coat. The finish coat wore off on the bottom but brush marks were still visible on the sides. The approach and the alinement were straight. The inlet was rounded. The readings were taken during flood flows, the discharge being rated by current meter. (*Transactions*, ASCE, Vol. 93, 1929, p. 1588.)

Curve 7. Lining finished with a brush coat. The finish coat wore off on the bottom but brush marks were still visible on the sides. The alinement was straight, the approach curved, and the inlet rounded. The readings were taken during flood flows, the discharge being rated by current meter. (References same as for curve 6.)

Curve 8. Tests by E. Kemler involving observations on 1,500 to 2,000 diameters of brass pipe 0.103 to 5.0 in. in diameter. The Nikuradse tests, indicated as plotted points (curve 11, Fig. 19), were included in the data that produced the Kemler curves. *Transactions*, A.S.M.E., Hydraulics Section, Vol. 55, 1933, p. 7–32.)

Curve 9. Hand-troweled cement finish; discharge rated by current meter placed in the tailrace.

DATA ON LARGE CONCRETE PIPE

Description and literary reference (8)
There were seven horizontal curves and two large vertical curves. The tests are reliable. (Reference same as for curve 1.) *Curve 10.* Some rough spots remained on the surface after the steel forms were removed. This was a poor test, no account being taken of the change in size and shape of the cross section. The alinement was irregular, with six horizontal bends and two vertical bends. The estimated error in the results is ±15%. (Letter dated March 5, 1931, from the Pacific Gas and Electric Company, San Francisco, Calif.) *Curve 11.* Smooth pipe; the plotted points designated curve 11 in Fig. 19 denote tests by J. Nikuradse on brass pipe from 0.033 in. in diameter to 3.28 in. in diameter. The equation through these points is good for extrapolation from $R = 30^6$ to 10^8. (*Forschungsheft 356*, Verein Deutscher Ingenieure, 1932, p. 30.) *Curve 12.* Steel forms were used and the concrete was rubbed with carborundum brick. The discharge was rated by color movement. The line included five bends on 800-ft radii, with short tangents between. This is equivalent to a curve length of 3,060 ft and a radius of 1,665 ft ($r/D = 92$). The bend loss (±5%) and surge tank losses were not considered. The data used were taken from a curve passing through forty-two observations. (*Bulletin No. 852*, U.S.D.A., Washington, D. C., 1924, p. 83; supplemented by correspondence with the Ontario Power Company in June, 1931, and April, 1935, including a map and a profile.) *Curve 13.* Hand-troweled cement finish; discharge rated by current meter placed in the tailrace. This was a very reliable test. The pipe was straight, free from bends and entrance losses, and was equipped with three excellent mercury monometers. The line was built in 1917. (References same as for curve 1.) *Curve 14.* Smooth cement surface; generator rating of discharge. It is reported that losses were high because of underestimated intake losses and poor location of the lower piezometer. The writers estimate that these factors make the plotted points ±7% high. (References same as for curve 1.) *Curve 15.* On this test oiled forms were used, and a neat cement brush coat. The lining was not smooth. The pipe was probably new at the time of the test and the discharge was rated by a weir below the plant. Surge chamber losses were neglected. The regained velocity head loss h_v was assumed equal to the entrance losses. The invert was placed by hand without forms and it presented a rather rough, uneven appearance. (*Proceedings*, June, 1923, Convention, Pacific Coast Electrical Assn., p. 139; *Engineering News-Record*, October 11, 1923, p. 598; and Library Data File, U.S.B.R., 91–241.) *Curve 16.* Surface originally smooth, had become somewhat eroded in 4 years. Discharge was rated by color movement. The line was pre-cast in 4-ft lengths in oiled steel forms. Joints were smooth. The water flowing in this line contains sharp basalt particles which have eroded the bottom of the intake like a sandblast. (*Bulletin No. 852*, U.S.D.A., Washington, D. C., 1924, p. 51). The water enters the pipe in a very turbulent state and the erosion extends 150 ft from the intake. (*Bulletin No. 852*, U.S.D.A., Washington, D. C., 1924, p. 36; also drawing No. 33.19(*b*) in the Denver office of U.S.B.R.) *Curve 17.* Use of steel forms in place have resulted in rough joints but with a smooth surface between joints. The line contains a sharp 90° bend and two slight vertical bends in the reach measured. About nineteen diameters upstream there is the last of two bends and constrictions resulting from repair work. The exit head loss was ignored and the results are not consistent. The concrete joints protrude as much as 0.15 ft in places. The effect of the bends was not considered in the computations. (*Bulletin No. 852*, U.S.D.A., Washington, D. C., 1924, p. 81; *Transactions*, ASCE, Vol. 73, September, 1911, pp. 399 and 460; "Catskill Water Supply of New York," by Lazarus White, John Wiley & Sons., Inc., New York, N. Y., 1913, pp. 66 and 73; and *Engineering Record*, January 1, 1910, p. 26; September 17, 1910, p. 312; March 11, 1911, p. 279; and February 28, 1914, p. 240). *Curve 18.* Surface polished. Friction values taken from curve prepared by E. W. Spies. ("Turbulente Stromungon in geraden und gekrummten glotten Rohrleitungen bei hohen Reynolds' schen Zahlen," by Romano Gregorig, Dipl. Masch-Ingenieure, Eidgenossichen Technischen Hochschule in Zurich, No. 695, 1933., D. F. 91–26.) *Curve 19.* Pre-cast in oiled steel forms with 12-ft lengths. The joints were smooth and carefully laid. Discharge was rated by venturi meter. In the first test there were twenty-nine bends; in the second, the line was slightly sinuous; and, in the third, the line was nearly straight. (*Engineering News-Record*, May 28, 1925, p. 897.) *Curve 20.* Smooth surface; discharge rated by color movement. The line was quite straight in horizontal alinement. Vertical curves were long and gentle. The reach includes five 6-in. valves and three manholes. (*Bulletin No. 852*, U.S.D.A., Washington, D. C., 1924, p. 35.) *Curve 21.* Smooth surface. Sections pre-cast in 6-ft steel forms. Discharge was rated by color movement. The alinement was straight. The reach includes eight 6-in. valves, two 6-in. blowoffs, and four 12-in. by 14-in. manholes. The inside surface was painted with a rich cement grout. (*Bulletin No. 852*, U.S.D.A., Washington, D. C., 1924, p. 41.) *Curve 22.* Smooth surface; discharge rated by water meter. The conduit in the concrete was formed by a 36.5-ft length of smooth straight rubber hose. (*Technical Memorandum No. 339*, U.S.B.R., Denver, Colo., June 15, 1933.) *Curve 23.* Sections pre-cast in oiled steel forms, 4 ft long, steam cured; discharge rated by color movement. The surface is reported as "unusually smooth" but for about half the line is curved gently and no allowance has been made for this curvature in the computations. (*Bulletin No. 852*, U.S.D.A., Washington, D. C., 1924, p. 39.) *Curve 24.* Surface formed by troweled gunite. (References same as for curve 18.) *Curve 25.* Use of steel forms in place has resulted in rough joints but with a smooth surface between joints. The line contains one sharp 90° bend in the reach measured. The concrete was poured against oiled steel forms but the joints were not smoothed. (*Bulletin No. 852*, U.S.D.A., Washington, D. C., 1924, p. 82; *Engineering Record*, April 2, 1910, p. 460.) *Curve 26.* Smooth steel forms were used by the Tennessee Valley Authority (TVA) on the Apalachia Tunnel. Discharge was rated by current meter. The tunnel consists of three types of lining—concrete, steel, and unlined rock. The concrete section contains four long radius bends in the test section. The entire tunnel has a total of six bends. *Curve 27.* The solid line marked "curve 27" in Fig. 19(*b*) was plotted from experimental data. *Curve 28.* This curve in Fig. 19(*b*) represents the von Kármán-Nikuradse equation: $1/f^{0.5} = 0.8 + 2 \log(R f^{0.5})$. *Curve 29.* Extrapolated from data for the 13-ft steel penstock tests at Boulder Dam, introducing the value of $f = 0.0177$ in the formula: $\frac{1}{f^{0.5}} = -2 \log \left(\frac{K}{7.4\, r} + \frac{2.51}{R\, f^{0.5}} \right)$. This formula was derived by C. F. Colebrook and C. M. White to cover the transition from smooth pipe to rough pipe ($K/r = 0.00134$).

of surface, and estimated reliability of each test. Also listed in Table 14 are the references to published papers and bulletins on original work from which the information in Fig. 19 was obtained. For the sake of clarity, the actual test points have been omitted. In most cases these did not fall directly on the line as drawn but fell to both sides with considerable variation in some cases. The lines represent average values.

Superimposed in Fig. 19(*a*) are the results of the Nikuradse laboratory tests on smooth brass pipe and on brass pipe artificially roughened with sand; the Kemler boundary of tests on smooth brass pipe; and a line representing the von Kármán and Nikuradse equation for friction coefficients in smooth pipes for large Reynolds' numbers. Also included in Fig. 19 (designated as curve 26) are the results on the concrete-lined section of the Apalachia Tunnel tests. These were obtained from Fig. 10(*a*).

In looking over this graphical presentation of nearly all the apparently reliable hydraulic friction tests of pressure conduits surfaced with concrete and including pipes from 1 in. to 18 ft in diameter, one is first impressed by the wide range of coefficients found at any given Reynolds' number. For example, at a Reynolds' number of 10,000,000, the coefficient varied from 0.008 in the Ontario Power Company test (curve 12) to about 0.0125 in the Apalachia tests (curve 26)—a range of about 25% from the mean value. Whether this variation represented a real difference in the character of the concrete surfaces in the two cases or whether it represented errors in the measurement of hydraulic losses or in computations made to allow for bend or entrance effects is not known. The concrete surface of the Ontario conduit is known to have been exceptionally smooth, but without a quantitative measure of the roughness, such as has been developed by mechanical engineers to measure the smoothness of bearings, it would appear useless to speculate on the reasons for the differences shown. In the circumstances, it would appear necessary for hydraulic engineers, in their computations, to allow for the extreme range of possible friction coefficients. Since this is an economically wasteful procedure, the need for the development of some quantitative laboratory methods of measuring roughness seems evident.

The second feature of the curves in Fig. 19 is the general upward slope of the lines showing the results of tests of individual pipes. An upward slope means that friction losses are increasing at a greater rate than the square of the velocity. This is in accord with the Colebrook and White[44] transition law for the change between smooth and rough surfaces and is indicated in Fig. 19 by the family of $\frac{K}{r}$-curves. Apparently in the larger pipe and at the economic velocities usual in hydraulic design, most flow falls within the transition classification. The plotted data should serve as a warning that, where large diversion tunnels are to be designed to function at high velocities, friction losses may prove to be greater than would be anticipated from low velocity tests. The writers would appreciate comments from the authors on this characteristic of the data presented in the paper.

[44] "Turbulent Flow in Pipes, with Particular Reference to the Transition Region Between the Smooth and Rough Pipe Laws," by C. F. Colebrook, *Journal*, Inst. C. E., London, Vol. 11, 1938–1939, pp. 133–136.